

# A Graphical Representation for the Stochastic Pi-Calculus

Andrew Phillips

Luca Cardelli

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Microsoft Research  
7 J J Thomson Avenue  
Cambridge, UK

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## Introduction

- Stochastic pi-calculus used to model and simulate a range of biological systems [Lecca and Priami, 2003, Priami et al., 2001, Regev et al., 2001]:
  - ❑ Able to model independent system components, which can be composed to predict emergent system behaviour.
  - ❑ Mathematical definition supports useful analysis techniques: type systems, behavioural equivalences, model checking.
  
- Mathematical syntax and semantics can limit accessibility to a wider audience:
  - ❑ Useful to present an alternative graphical view
  - ❑ Particularly welcomed by experimental systems biologists.

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## Outline

- Stochastic pi-calculus
- Graphical stochastic pi-calculus:
  - ❑ Graphical syntax
  - ❑ Graphical execution model
- Graphical biological examples:
  - ❑ Evolved gene network [Francois and Hakim, 2004]
  - ❑ Mapk signalling cascade [Huang and Ferrel, 1996]
- Automatic graph generation:
  - ❑ Encoding pi-calculus to an open graph syntax
  - ❑ Front end to a graphical simulator/debugger (ongoing)

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## The Stochastic Pi-Calculus (SPi)

Each channel  $x$  is associated with a stochastic rate given by  $rate(x)$

$\pi ::=$	$?x(\vec{m})$	Input
	$!x(\vec{n})$	Output
	$\tau_r$	Delay
$P, Q ::=$	$\pi_1.P_1 + \dots + \pi_N.P_N$	Choice
	$P_1 \mid \dots \mid P_N$	Parallel
	$\nu x P$	Restriction
	$X(\vec{n})$	Instance

$\Gamma ::= X_1(\vec{m}_1) \triangleq P_1, \dots, X_N(\vec{m}_N) \triangleq P_N$  Definitions,  $\text{fn}(P_i) \subseteq \vec{m}_i$

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# The SPiM Programming Language (v0.04)

$Dec$	$::=$	<b>new</b> $x\{\textcircled{r}\}:t$	Channel Declaration
		<b>type</b> $n = t$	Type Declaration
		<b>val</b> $m = v$	Value Declaration
		<b>run</b> $P$	Process Declaration
		<b>let</b> $D_1$ <b>and</b> $\dots$ <b>and</b> $D_N$	Definitions, $N \geq 1$
$D$	$::=$	$X(m_1, \dots, m_N) = P$	Definition, $N \geq 0$
$P$	$::=$	$()$	Null Process
		$(P_1 \mid \dots \mid P_M)$	Parallel, $M \geq 2$
		$X(v_1, \dots, v_N)$	Instantiation, $N \geq 0$
		$\pi\{; P\}$	Action
		<b>do</b> $\pi_1\{; P_1\}$ <b>or</b> $\dots$ <b>or</b> $\pi_M\{; P_M\}$	Choice, $M \geq 2$
		$(Dec_1 \dots Dec_N P)$	Declarations, $N \geq 0$
$\pi$	$::=$	<b>!</b> $x \{(v_1, \dots, v_N)\}$	Output, $N \geq 0$
		<b>?</b> $x \{(m_1, \dots, m_N)\}$	Input, $N \geq 0$
		<b>delay</b> $\textcircled{r}$	Delay

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# The Graphical Stochastic Pi-Calculus (GSPi)

A normal form for SPi, with each summation or guarded process as a definition:

$\pi ::=$	$?x(\vec{m})$	Input
	$!x(\vec{n})$	Output
	$\tau_r$	Delay
$P, Q ::=$	$P_1 \mid \dots \mid P_N$	Parallel
	$\nu x P$	Restriction
	$X(\vec{n})$	Instance

$\Gamma ::=$	$X(\vec{m}) \triangleq \nu x_1 \dots \nu x_M (\pi_1.X_1(\vec{n}_1) + \dots + \pi_N.X_N(\vec{n}_N))$	Summation
	$X(\vec{m}) \triangleq \nu x_1 \dots \nu x_M (X_1(\vec{n}_1) \mid \dots \mid X_N(\vec{n}_N))$	Composition

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## Graphical Representation: Definitions

- A collection of mutually recursive definitions:

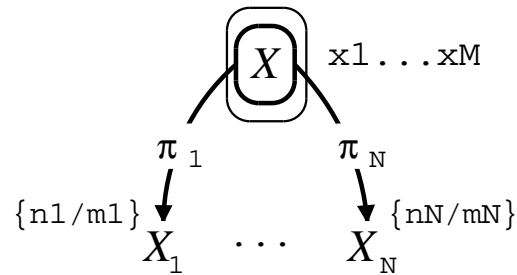
$$X_1(m_1) \triangleq C_1, \dots, X_N(m_N) \triangleq C_N$$

- Displayed as a directed graph with nodes  $X_1 \dots X_N$  and with edges between these nodes.
- Each definition  $X(m) \triangleq C$  displayed as a node  $X$  with zero or more edges to subsequent nodes .

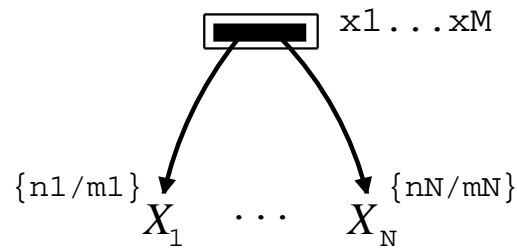
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## Graphical Representation: Definitions

$$X(\vec{m}) \triangleq \nu x_1 \dots \nu x_M (\pi_1.X_1(\vec{n}_1) + \dots + \pi_N.X_N(\vec{n}_N))$$



$$X(\vec{m}) \triangleq \nu x_1 \dots \nu x_M (X_1(\vec{n}_1) \mid \dots \mid X_N(\vec{n}_N))$$



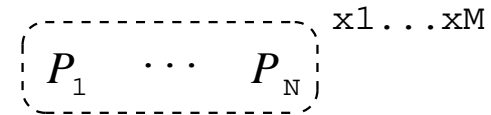


# Graphical Representation: Processes

$X(\vec{n})$

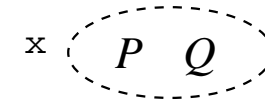


$\nu x_1 \dots \nu x_M (P_1 \mid \dots \mid P_N)$

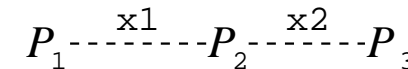
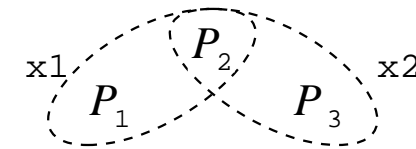


Restriction as Complexation:

A complex of  $P$  and  $Q$  modelled as a restriction  $\nu x (P \mid Q)$



$\nu x_1 \nu x_2 (P_1 \mid P_2 \mid P_3)$ ,  $x_1 \notin \text{fn}(P_3)$ ,  $x_2 \notin \text{fn}(P_1)$



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## Graphical Reduction: Execution Model

Reduction in SPi:

$$!x(\vec{n}).P + \Sigma \mid ?x(\vec{m}).Q + \Sigma' \xrightarrow{\text{rate}(x)} P \mid Q_{\{\vec{n}/\vec{m}\}} \quad (1)$$

$$\tau_r.P + \Sigma \xrightarrow{r} P \quad (2)$$

$$P \xrightarrow{r} P' \Rightarrow P \mid Q \xrightarrow{r} P' \mid Q \quad (3)$$

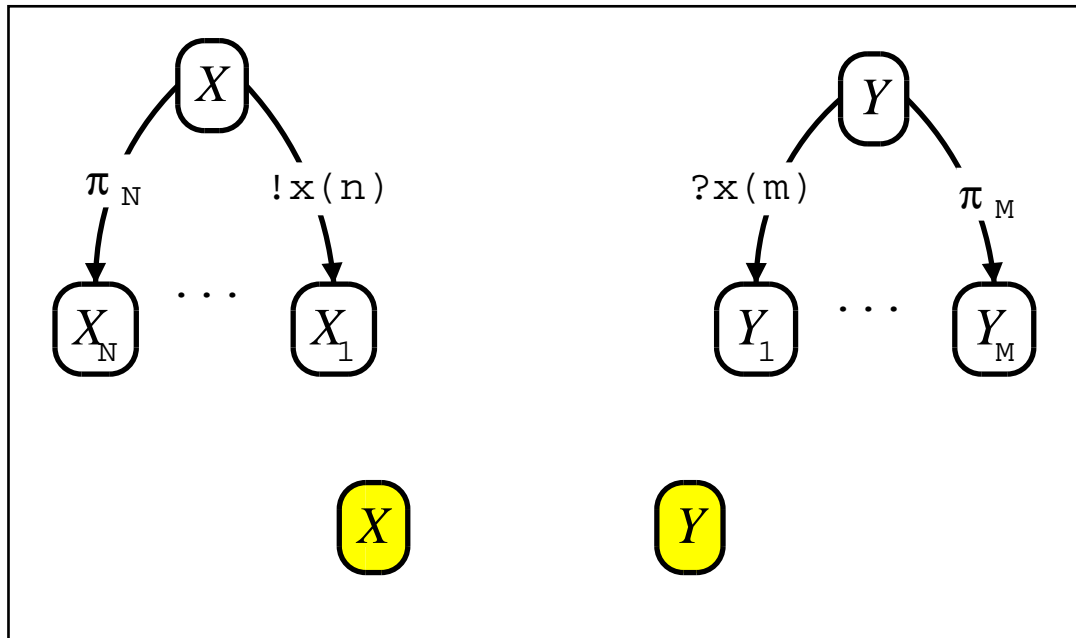
$$P \xrightarrow{r} P' \Rightarrow \nu x P \xrightarrow{r} \nu x P' \quad (4)$$

$$Q \equiv P \xrightarrow{r} P' \equiv Q' \Rightarrow Q \xrightarrow{r} Q' \quad (5)$$

Reduction in  $\text{GSPi} \subset \text{SPi}$ :

**Proposition 1.**  $\forall P \in \text{GSPi}. P \xrightarrow{r} P' \Rightarrow \exists P'' \in \text{GSPi}. P' \equiv P''$

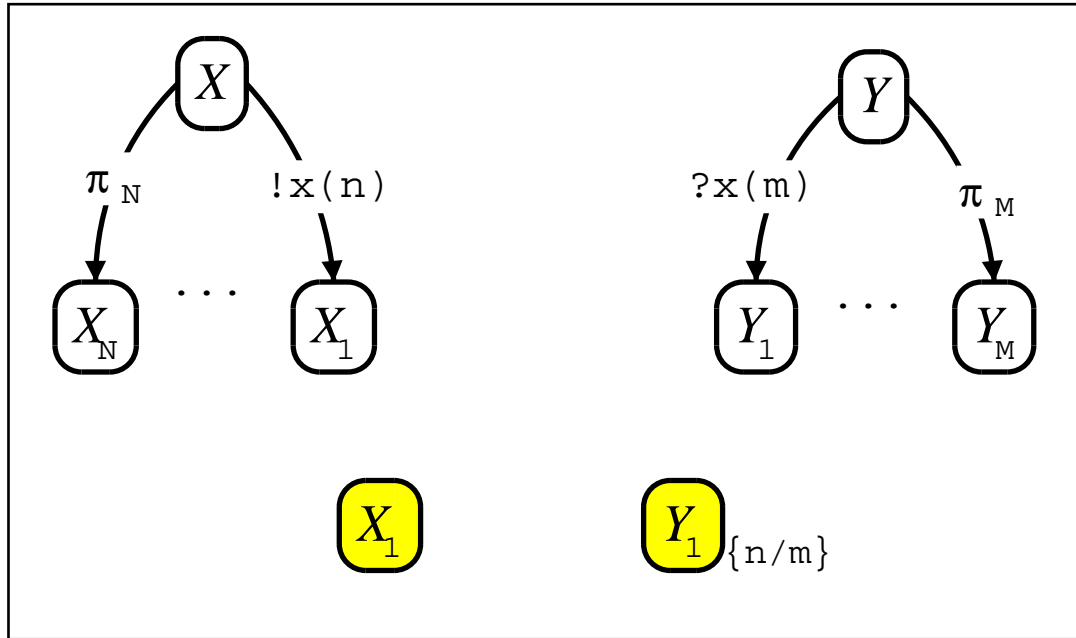
# Graphical Reduction: Communication



$$\underline{X(\vec{z}) \triangleq !x(\vec{n}).X_1(\vec{z}) + \dots + \pi_N.X_N(\vec{z})}, \quad \underline{Y \triangleq ?x(\vec{m}).Y_1(\vec{z}) + \dots + \pi_M.Y_M(\vec{z})}$$

$$\underline{X(\vec{z}) \mid Y(\vec{z})} \xrightarrow{\text{rate}(x)} \underline{X_1(\vec{z}) \mid Y_1(\vec{z})}_{\{\vec{n}/\vec{m}\}}$$

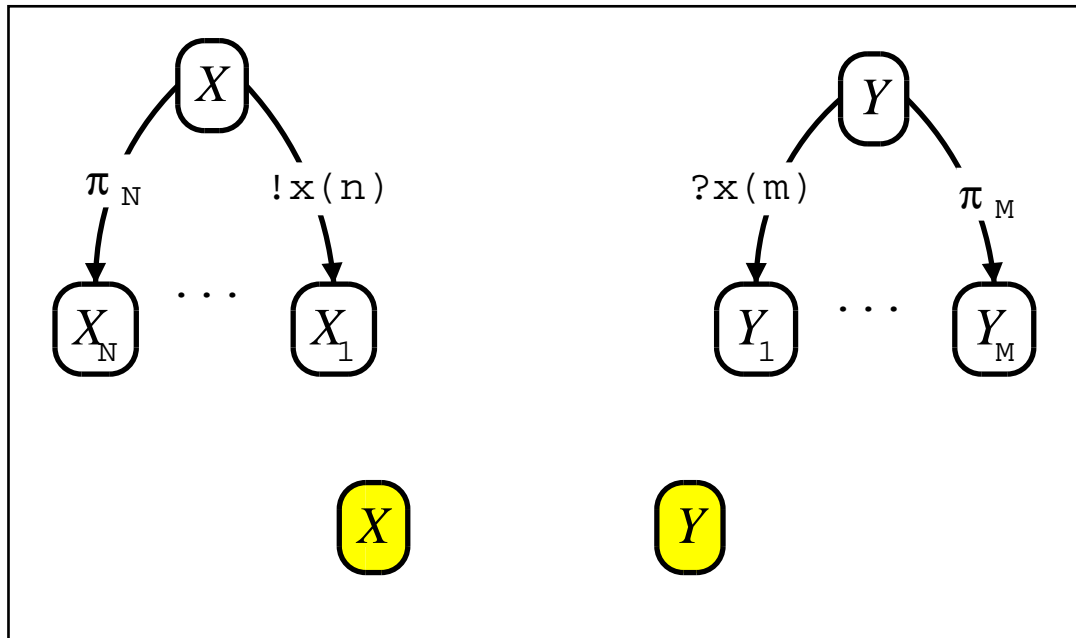
## Graphical Reduction: Communication



$$X(\vec{z}) \triangleq !x(\vec{n}).X_1(\vec{z}) + \dots + \pi_N.X_N(\vec{z}) , \quad Y \triangleq ?x(\vec{m}).Y_1(\vec{z}) + \dots + \pi_M.Y_M(\vec{z})$$

$$X(\vec{z}) \mid Y(\vec{z}) \xrightarrow{\text{rate}(x)} \underline{X_1(\vec{z}) \mid Y_1(\vec{z})_{\{n/m\}}}$$

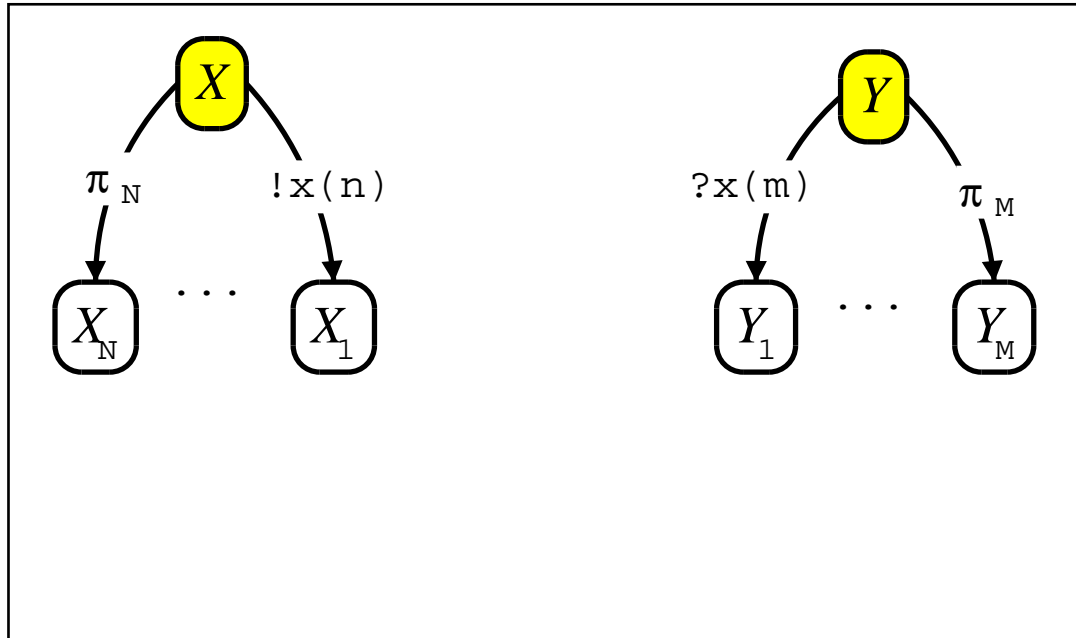
## Graphical Reduction: Communication



$$\underline{X(\vec{z}) \triangleq !x(\vec{n}).X_1(\vec{z}) + \dots + \pi_N.X_N(\vec{z})}, \quad \underline{Y \triangleq ?x(\vec{m}).Y_1(\vec{z}) + \dots + \pi_M.Y_M(\vec{z})}$$

$$\underline{X(\vec{z}) \mid Y(\vec{z})} \xrightarrow{\text{rate}(x)} \underline{X_1(\vec{z}) \mid Y_1(\vec{z})}_{\{\vec{n}/\vec{m}\}}$$

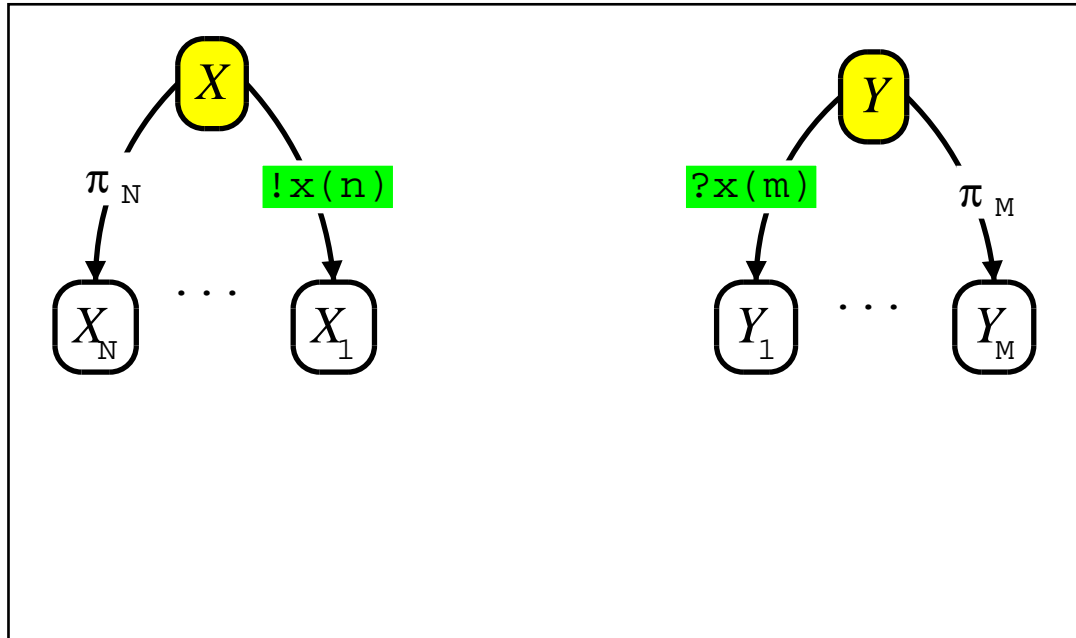
## Inline Graphical Reduction: Communication



$$X(\vec{z}) \triangleq !x(\vec{n}).X_1(\vec{z}) + \dots + \pi_N.X_N(\vec{z}) , \quad Y \triangleq ?x(\vec{m}).Y_1(\vec{z}) + \dots + \pi_M.Y_M(\vec{z})$$

$$\underline{X(\vec{z}) \mid Y(\vec{z})} \xrightarrow{\text{rate}(x)} X_1(\vec{z}) \mid Y_1(\vec{z})_{\{\vec{n}/\vec{m}\}}$$

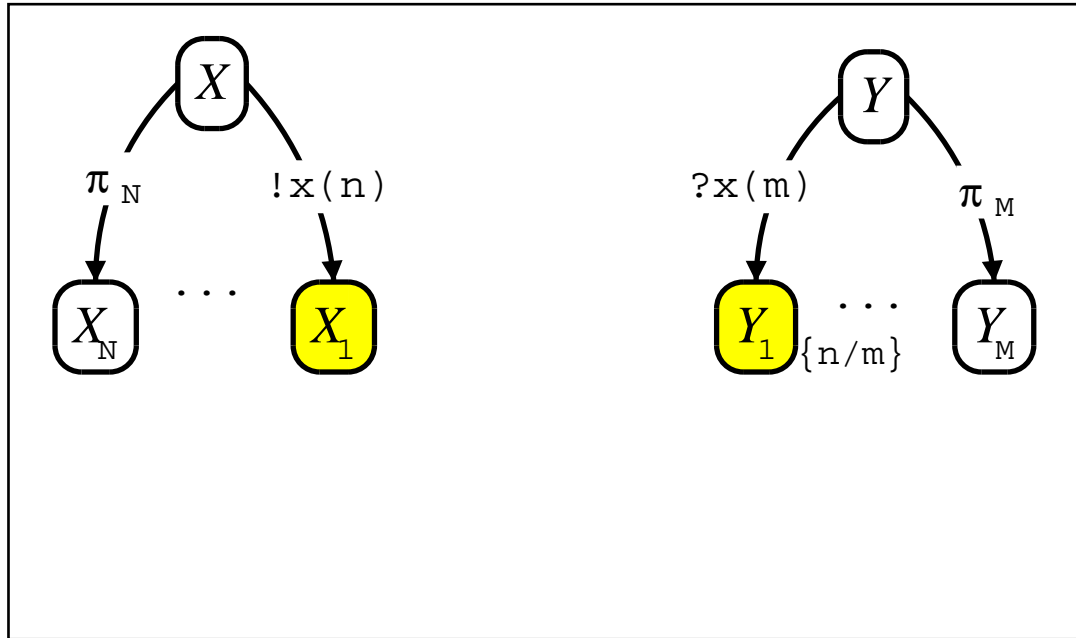
## Inline Graphical Reduction: Communication



$$X(\vec{z}) \triangleq !x(\vec{n}).X_1(\vec{z}) + \dots + \pi_N.X_N(\vec{z}) , \quad Y \triangleq ?x(\vec{m}).Y_1(\vec{z}) + \dots + \pi_M.Y_M(\vec{z})$$

$$\underline{X(\vec{z}) \mid Y(\vec{z})} \xrightarrow{\text{rate}(x)} X_1(\vec{z}) \mid Y_1(\vec{z})_{\{\vec{n}/\vec{m}\}}$$

## Inline Graphical Reduction: Communication



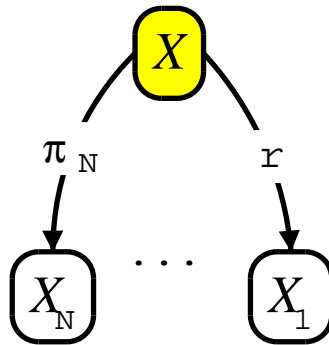
$$X(\vec{z}) \triangleq !x(\vec{n}).X_1(\vec{z}) + \dots + \pi_N.X_N(\vec{z}) , \quad Y \triangleq ?x(\vec{m}).Y_1(\vec{z}) + \dots + \pi_M.Y_M(\vec{z})$$

$$X(\vec{z}) \mid Y(\vec{z}) \xrightarrow{\text{rate}(x)} \underline{X_1(\vec{z}) \mid Y_1(\vec{z})_{\{\vec{n}/\vec{m}\}}}$$



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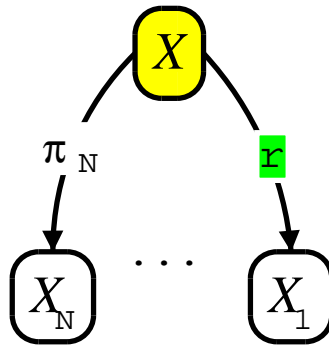
## Inline Graphical Reduction: Delay



$$\frac{X(\vec{z}) \triangleq \tau_r.X_1(\vec{z}) + \dots + \pi_N.X_N(\vec{z})}{\underline{X(\vec{z})} \xrightarrow{r} X_1(\vec{z})}$$

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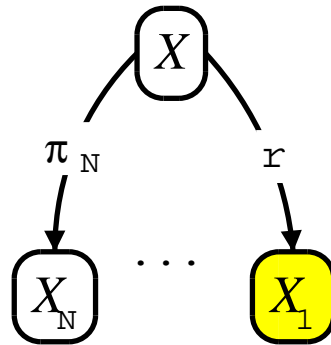
## Inline Graphical Reduction: Delay



$$\frac{X(\vec{z}) \triangleq \tau_r.X_1(\vec{z}) + \dots + \pi_N.X_N(\vec{z})}{\underline{X(\vec{z})} \xrightarrow{r} X_1(\vec{z})}$$

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## Inline Graphical Reduction: Delay



$$\frac{X(\vec{z}) \triangleq \tau_r.X_1(\vec{z}) + \dots + \pi_N.X_N(\vec{z})}{X(\vec{z}) \xrightarrow{r} \underline{X_1(\vec{z})}}$$

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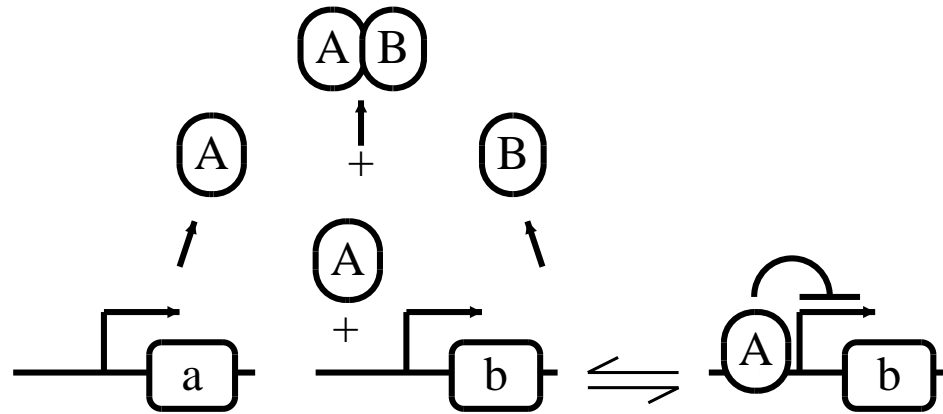
## Graphical Representation: Benefits

- Static graphical representation used to:
  - Clarify the connectivity between process definitions
  - Highlight cycles, which are key to many biological systems.
- Dynamic graphical representation used to:
  - Visualise the execution trace of a model
  - Clarify the overall system function
  - Graphically debug pi-calculus code.

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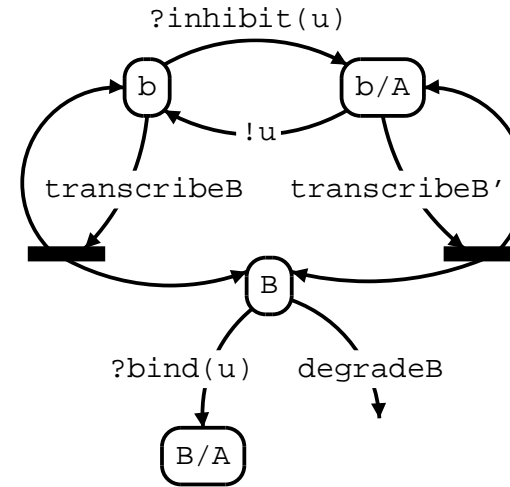
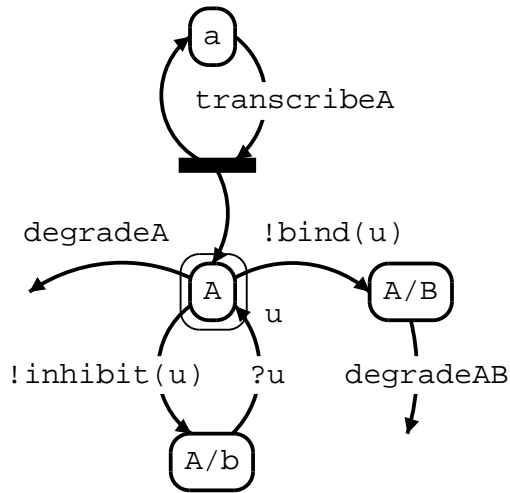
## Evolved Gene Network [Francois and Hakim, 2004]

- Gene networks are evolved in silico to perform specific functions, e.g.:



- Genes *a* and *b* can produce proteins *A* and *B* respectively:
- ❑ *A* and *B* can bind irreversibly to produce *AB*, which eventually degrades.
  - ❑ *A* can also bind reversibly to gene *b*, slowing the transcription of *B*.
- What is the function of this system?

# Evolved Gene Network: Definitions



$$a(\vec{z}) \triangleq \tau_{\text{transcribeA}}.(A() \mid a())$$

$$A(\vec{z}) \triangleq \nu u (\tau_{\text{degradeA}} + !\text{bind}(u).AB(u) + !\text{inhibit}(u).Ab(u))$$

$$Ab(u) \triangleq ?u.A()$$

$$AB(u) \triangleq \tau_{\text{degradeAB}}$$

$$b(\vec{z}) \triangleq \tau_{\text{transcribeB}}.(B() \mid b())$$

$$+ ?\text{inhibit}(u).bA(u)$$

$$bA(u) \triangleq \tau_{\text{transcribeB'}}.(B() \mid bA(u)) + !u.b()$$

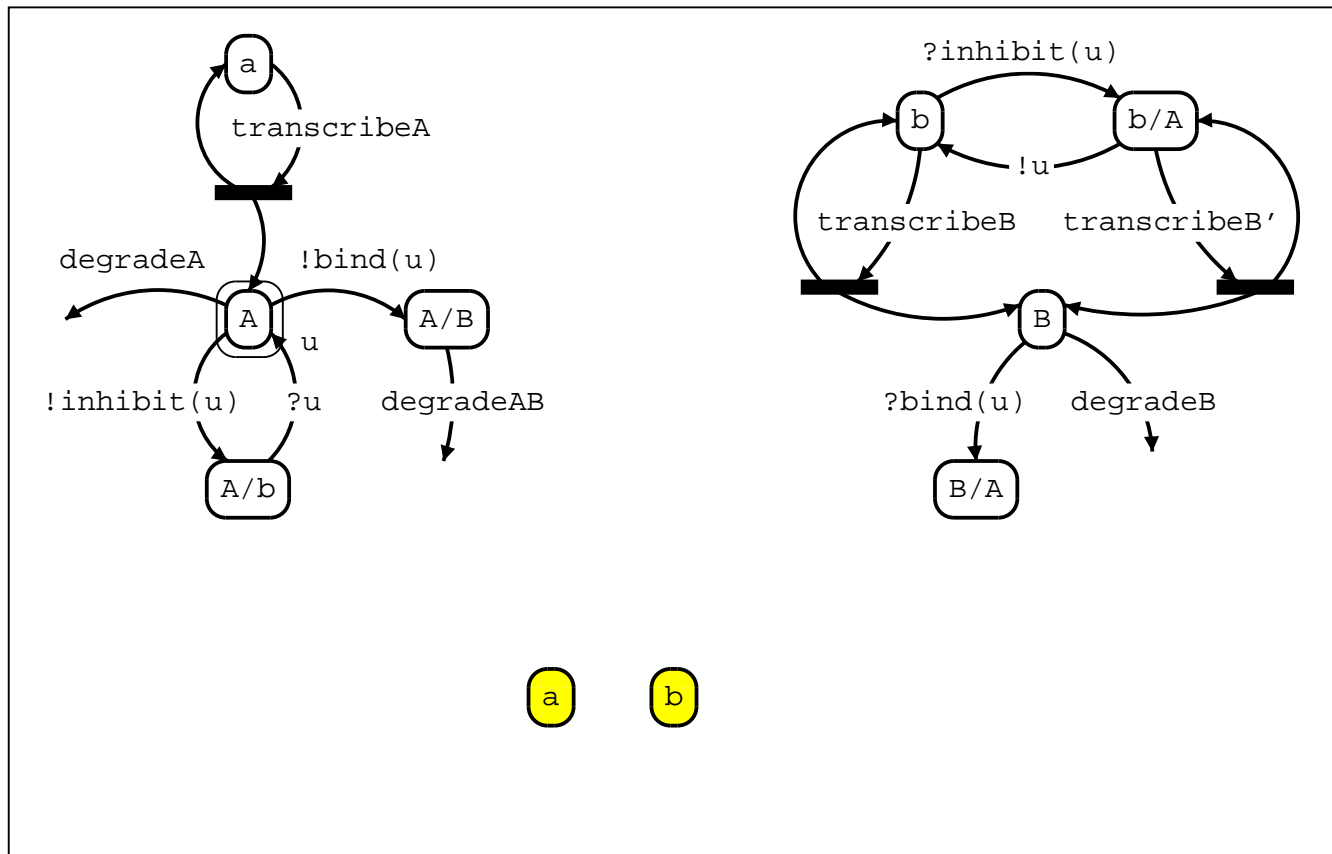
$$B(\vec{z}) \triangleq \tau_{\text{degradeB}} + ?\text{bind}(u).BA(u)$$

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## Evolved Gene Network: SPiM Code

```
let a() = delay@transcribeA; ( A() | a() )
and A() = (
  new u@0.42:chan
  do delay@degradeA
  or !bind; A_B()
  or !inhibit(u); A_b(u))
and A_b(u:chan) = ?u; A()
and A_B() = delay@degradeAB
let b() =
  do delay@transcribeB; ( B() | b() )
  or ?inhibit(u); b_A(u)
and b_A(u:chan) =
  do !u; b()
  or delay@transcribeB'; B(); b_A(u)
and B() = do delay@degradeB or ?bind
run (a() | b())
```

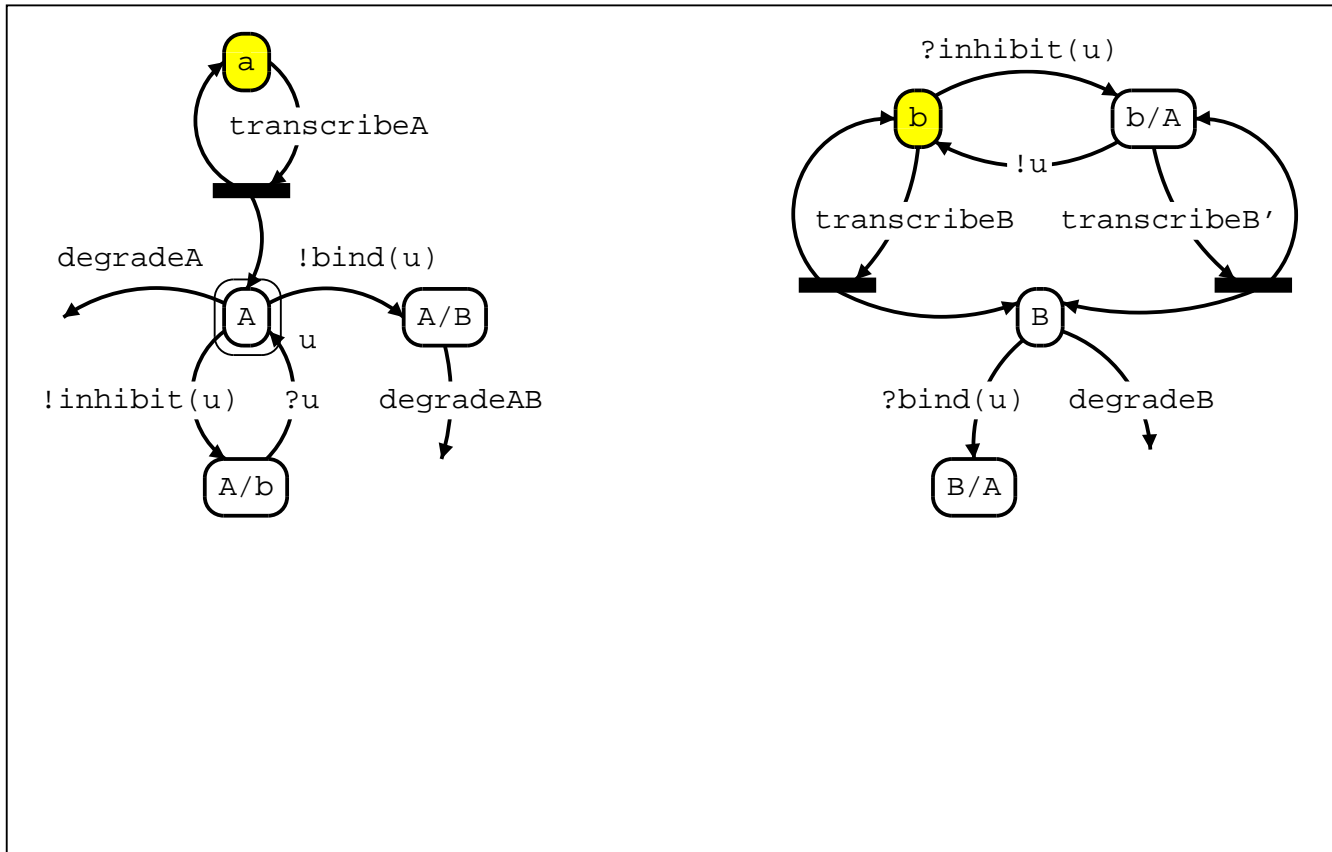
# Evolved Gene Network



Initially there is one copy of each gene, *a* and *b*

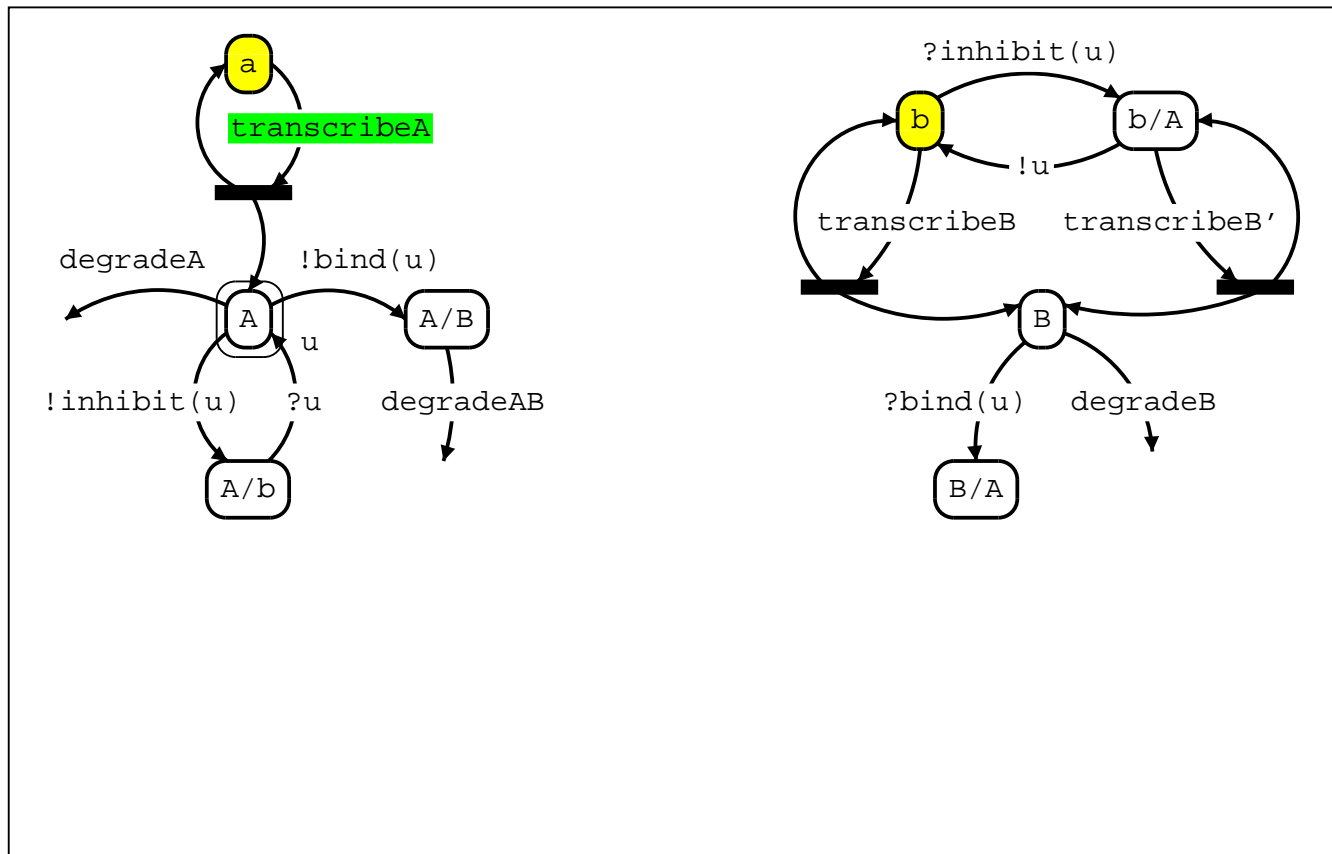


# Evolved Gene Network



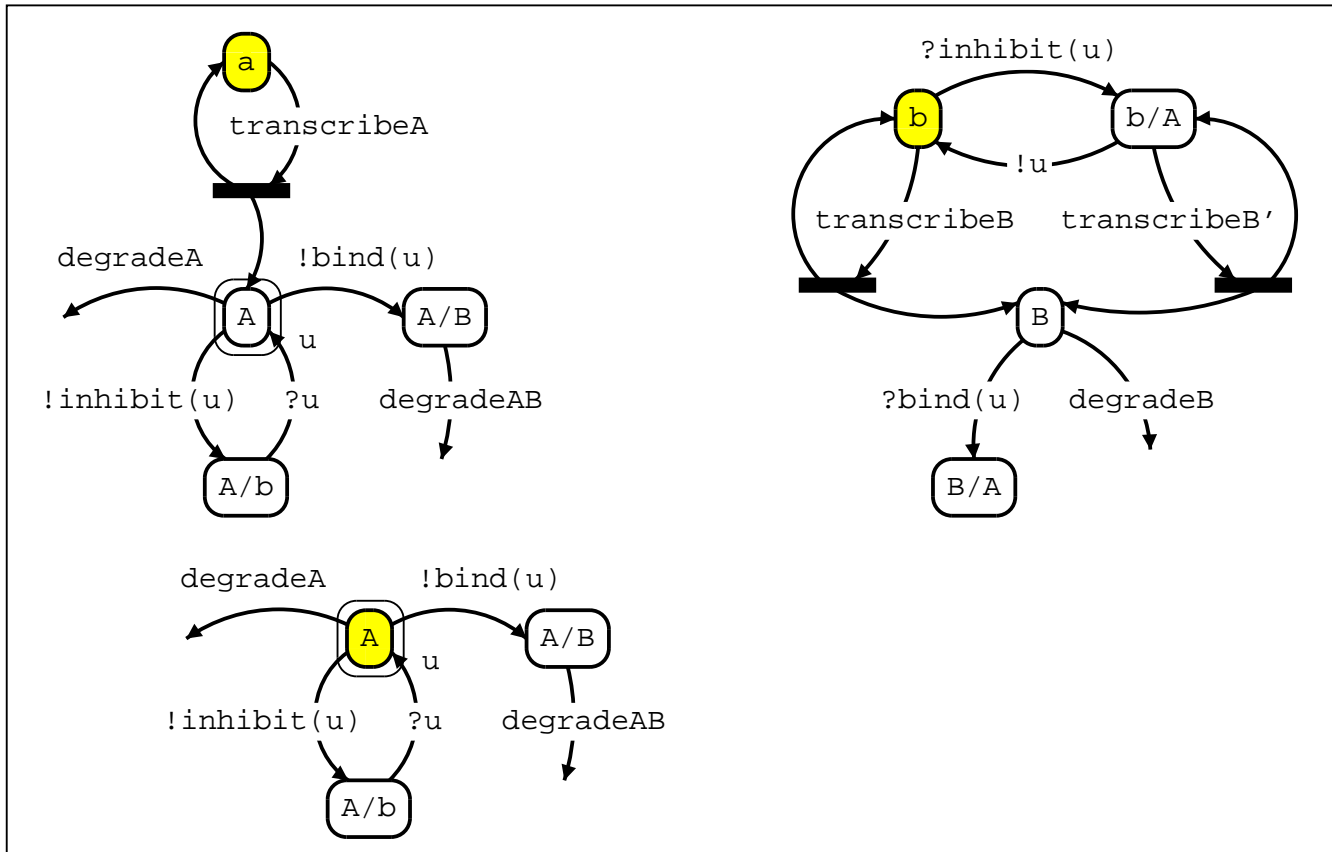
Represent the behaviour of each gene as a separate graph

# Evolved Gene Network



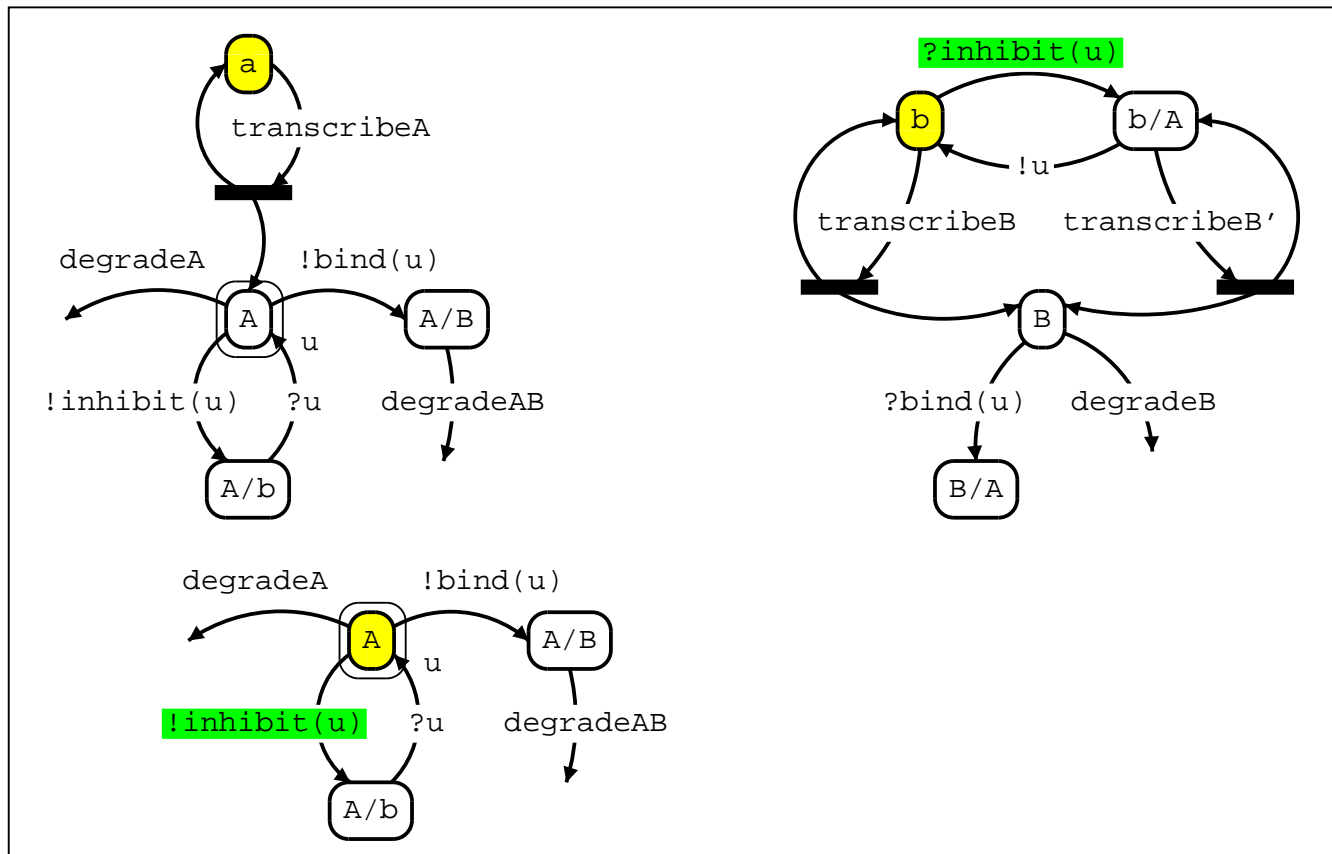
Gene  $a$  can transcribe a new protein  $A$  at rate  $transcribeA$

# Evolved Gene Network



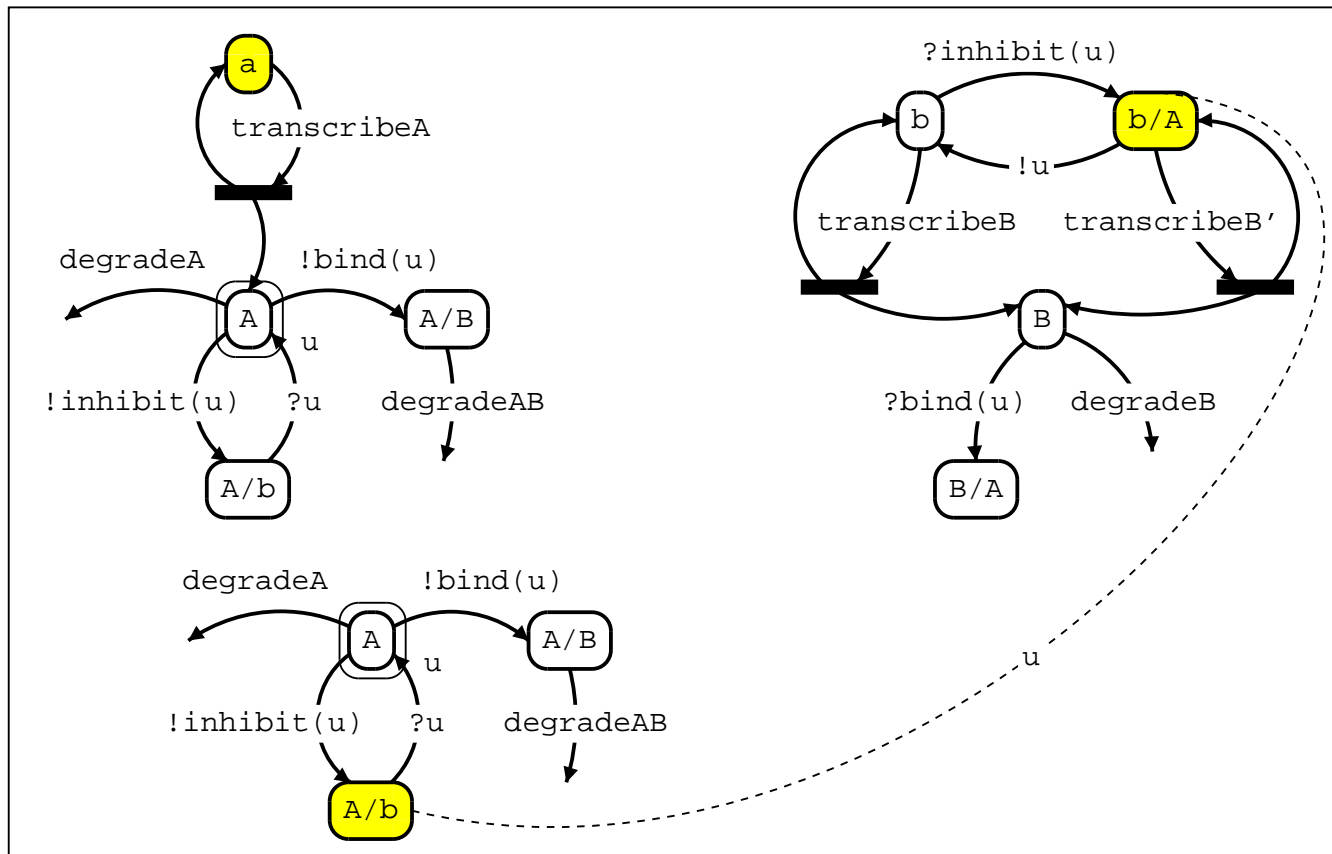
A new protein *A* is transcribed

# Evolved Gene Network



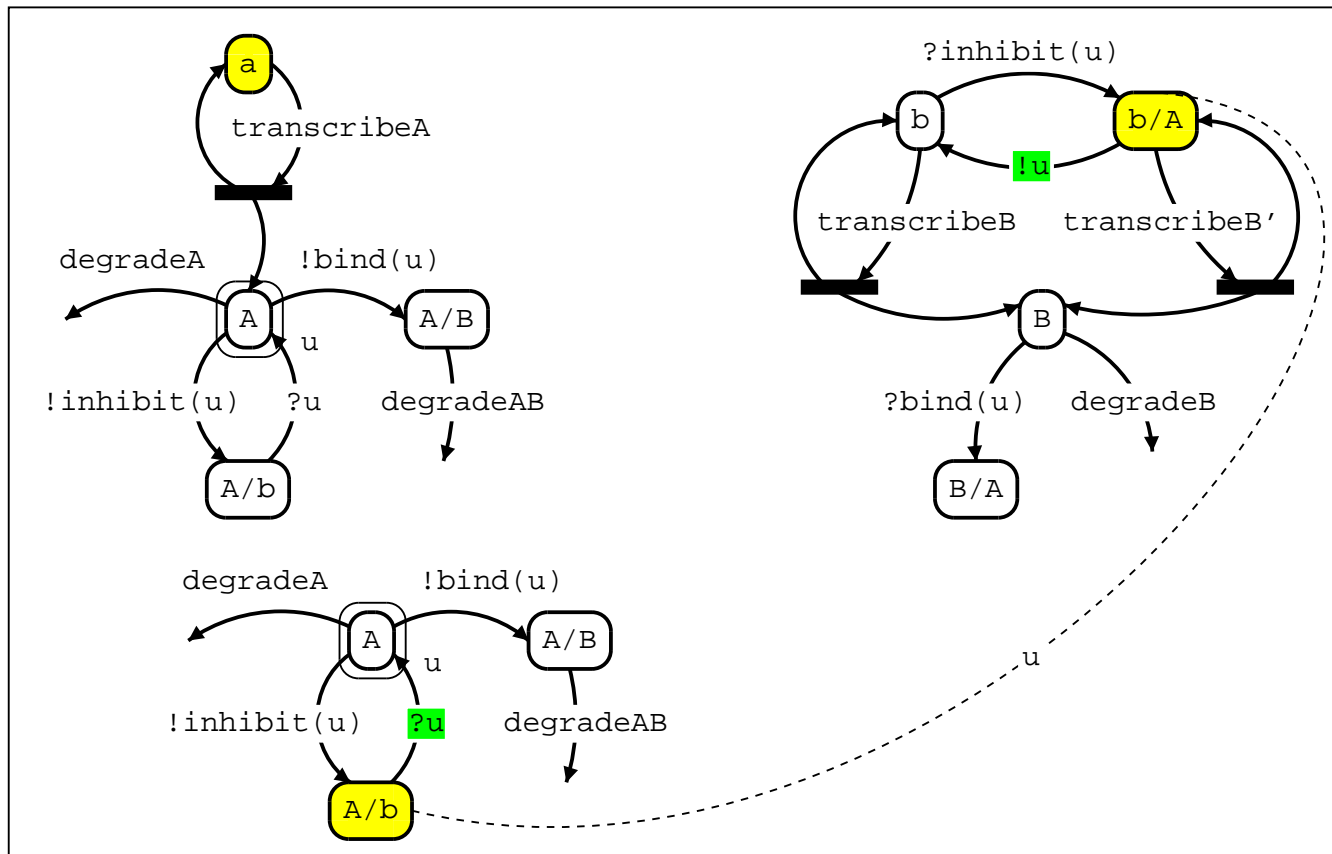
Protein *A* can bind to gene *b* to inhibit production of protein *B*

# Evolved Gene Network



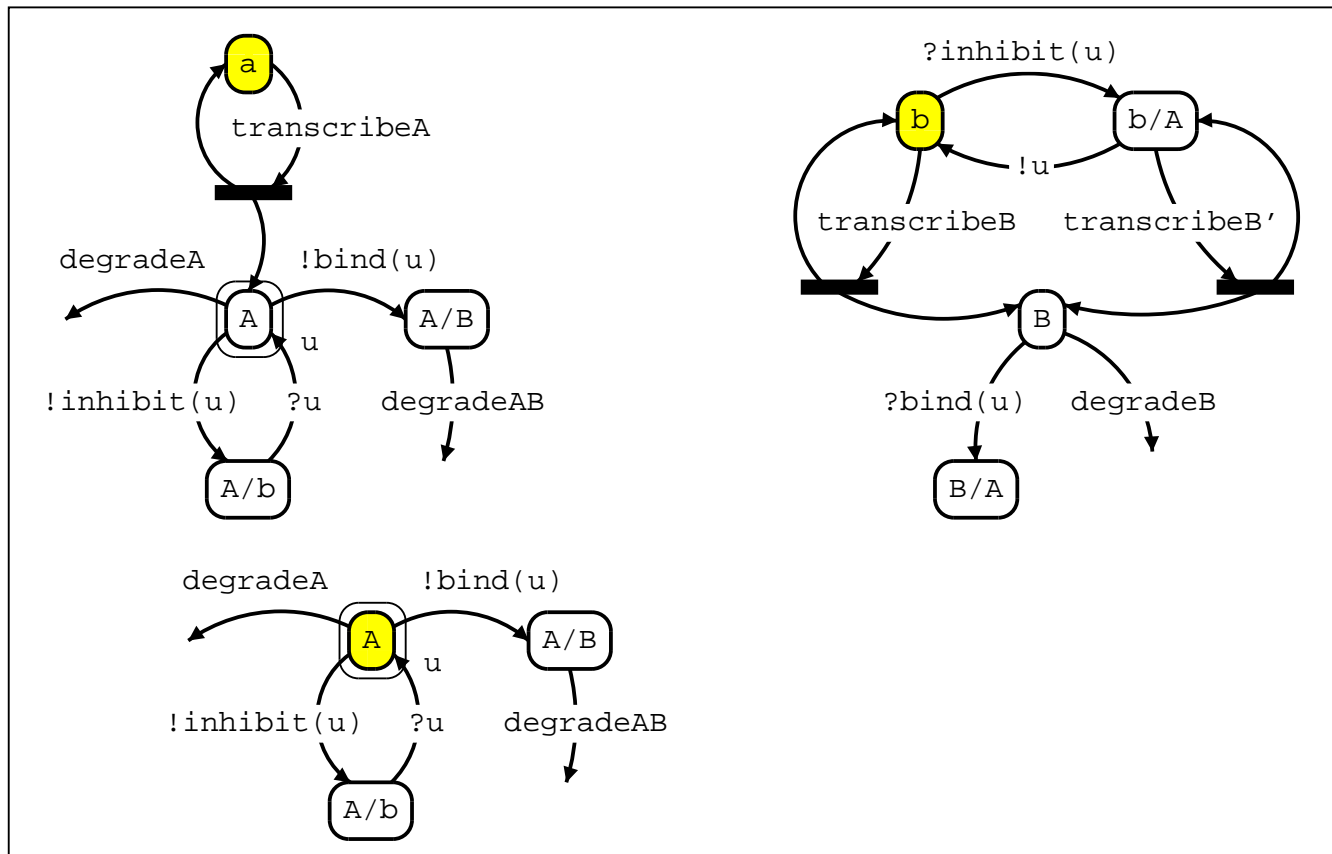
Protein *A* is bound to gene *b* by the private channel *u*

# Evolved Gene Network



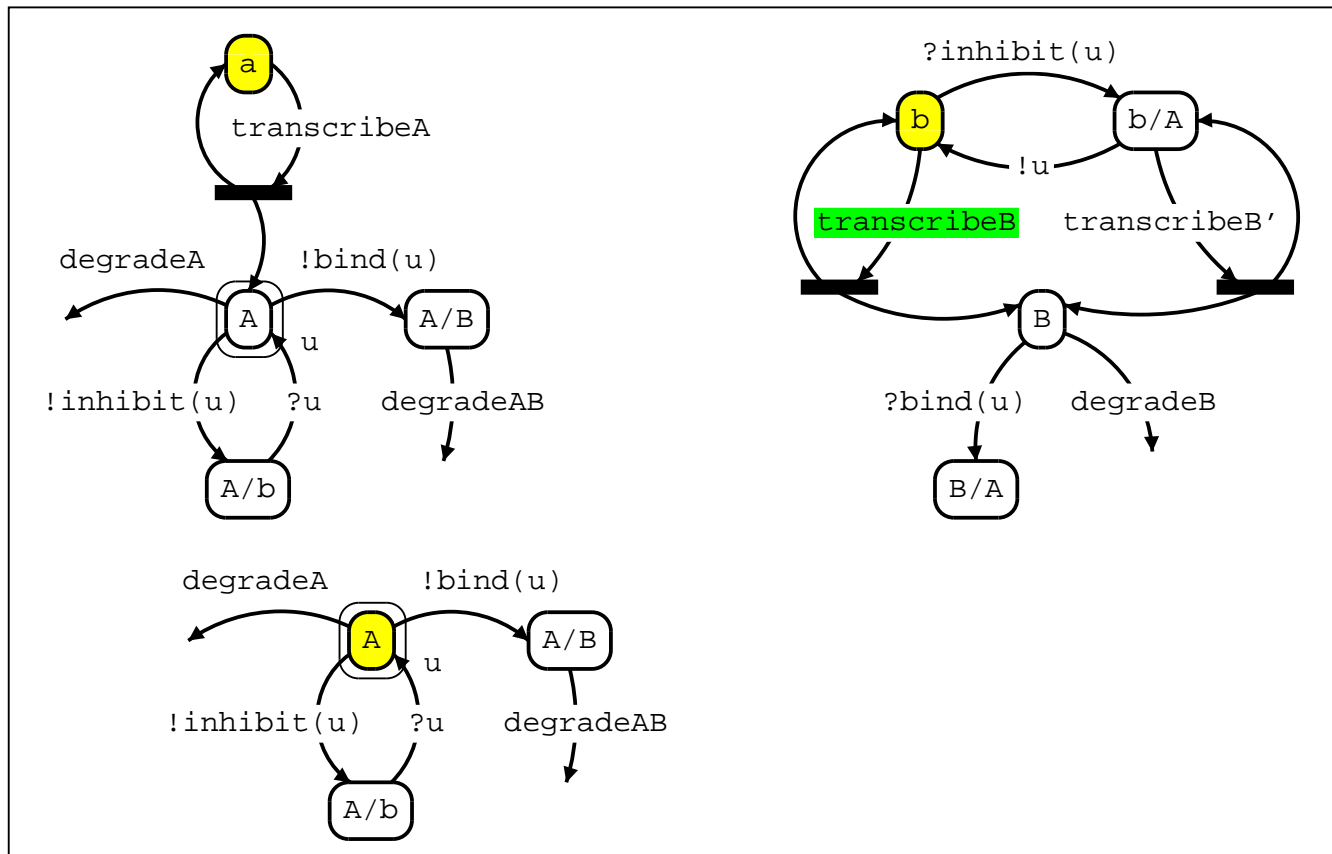
Protein *A* can unbind from gene *b* using channel *u*

# Evolved Gene Network



Protein *A* is no longer bound to gene *b*

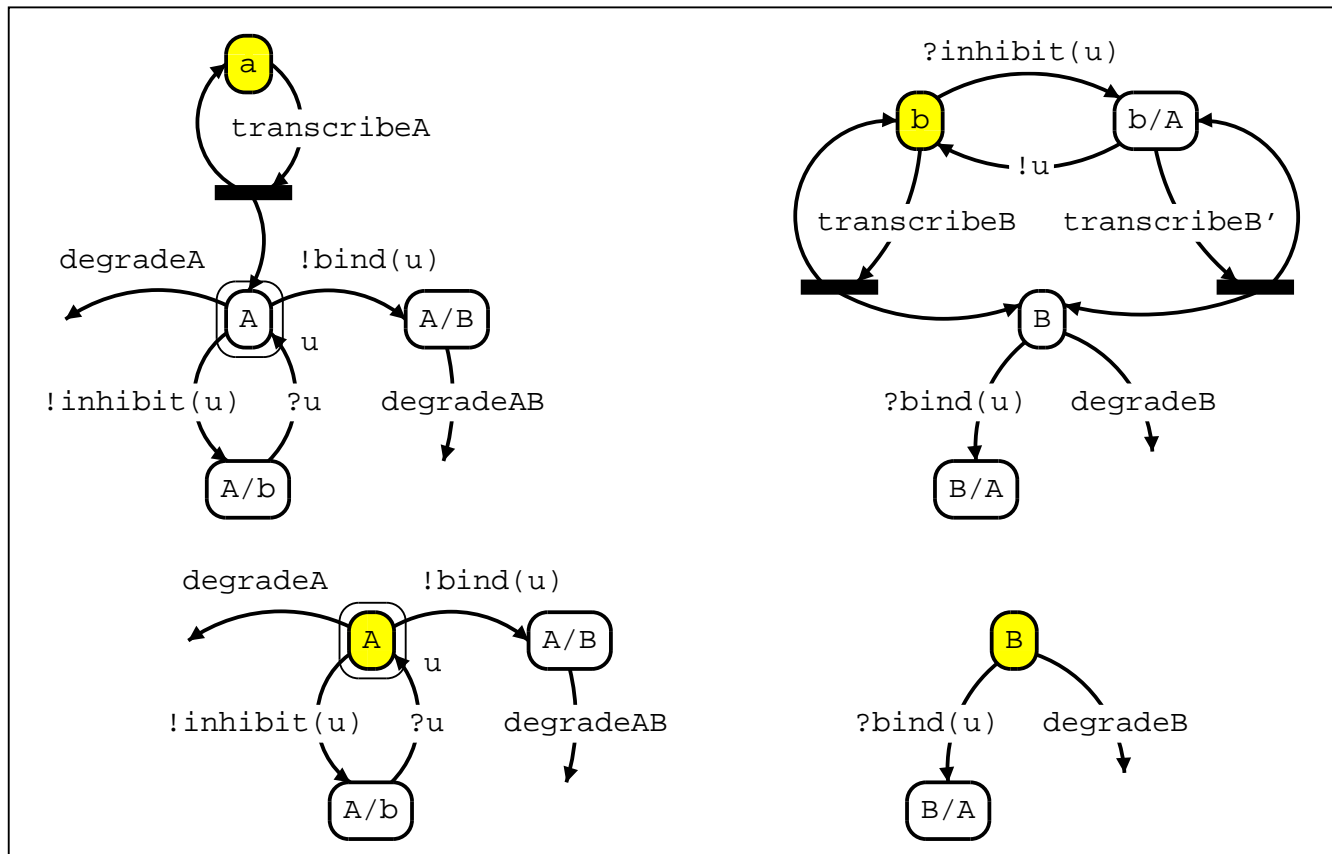
# Evolved Gene Network



Gene *b* can transcribe a new protein *B* with rate *transcribeB*

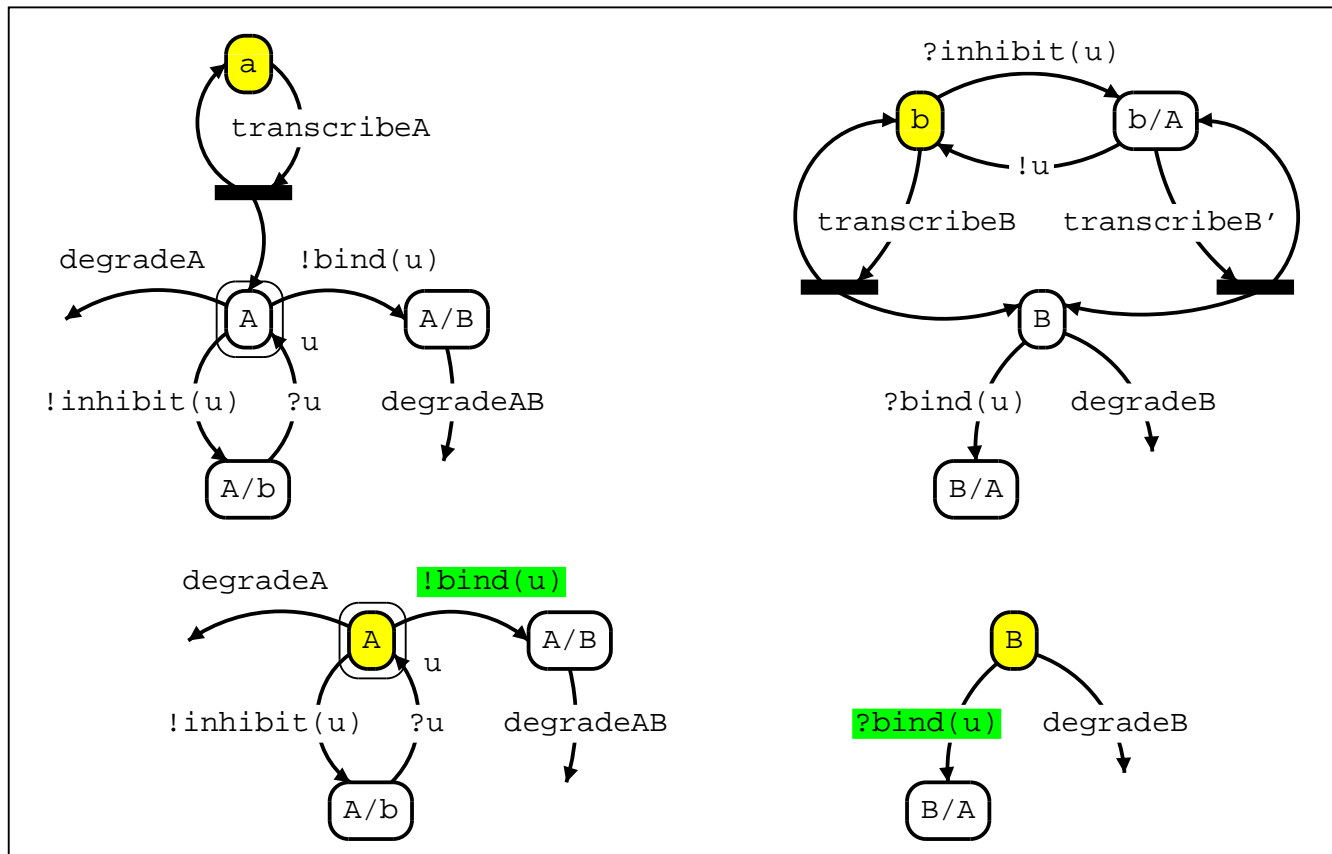


# Evolved Gene Network



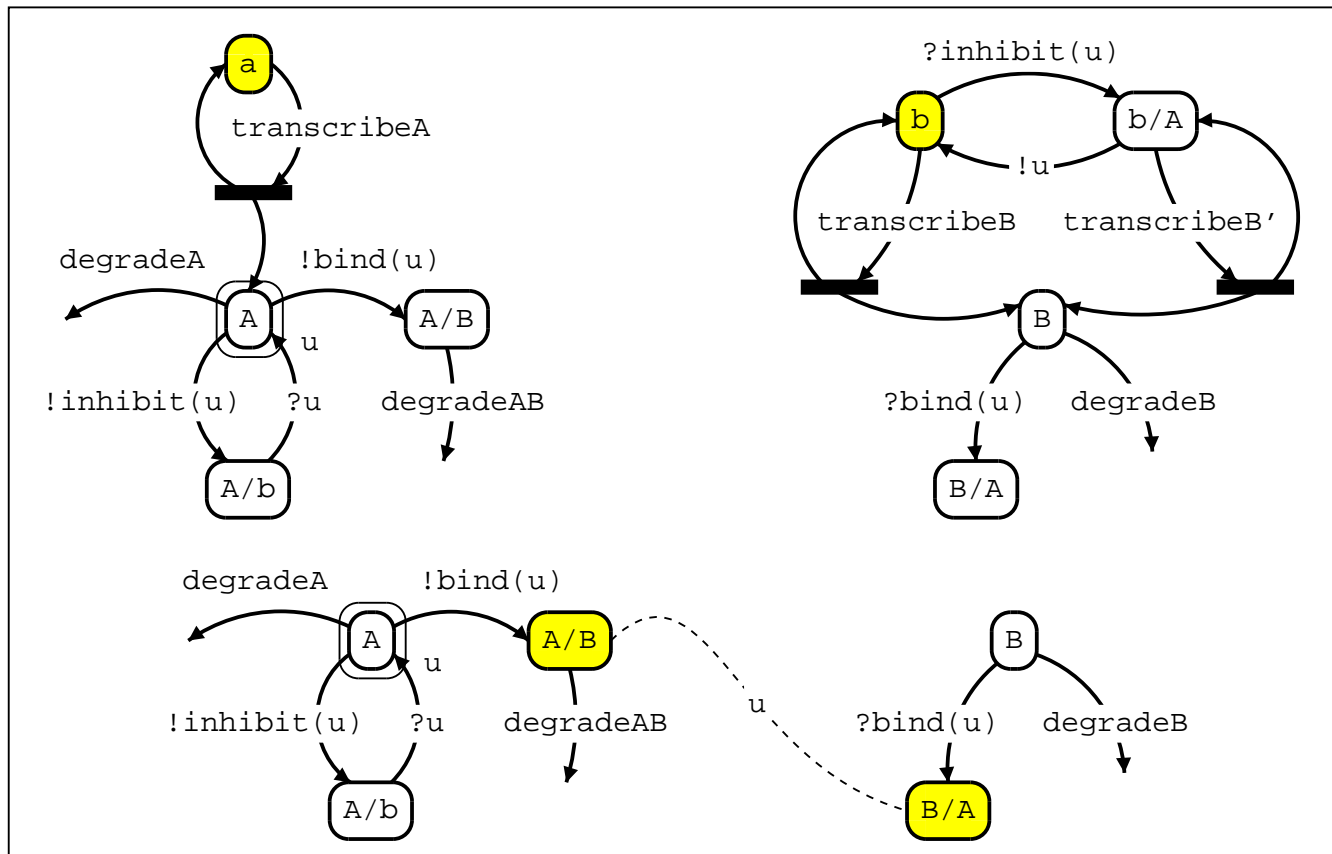
A new protein *B* is transcribed

# Evolved Gene Network



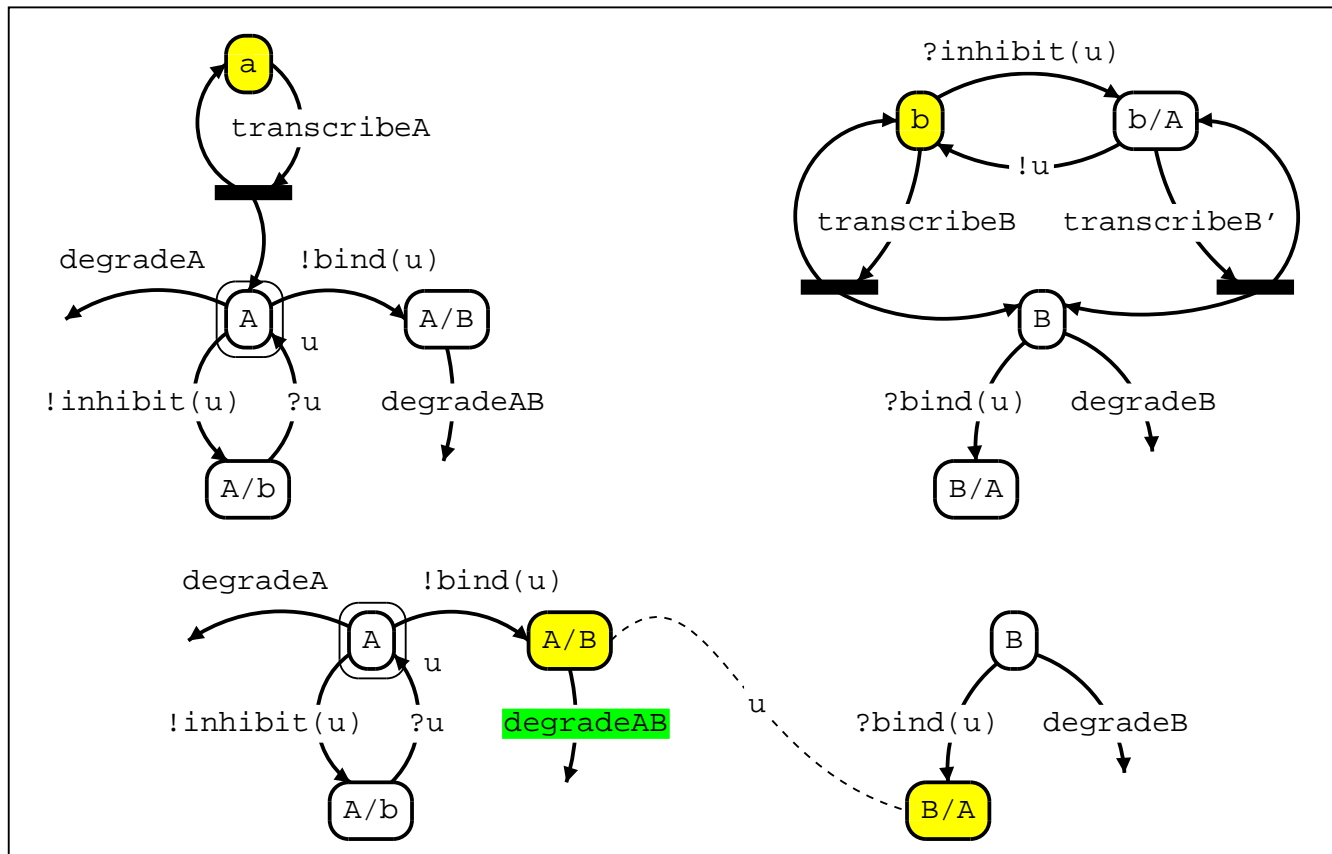
Protein *A* can bind with protein *B*

# Evolved Gene Network



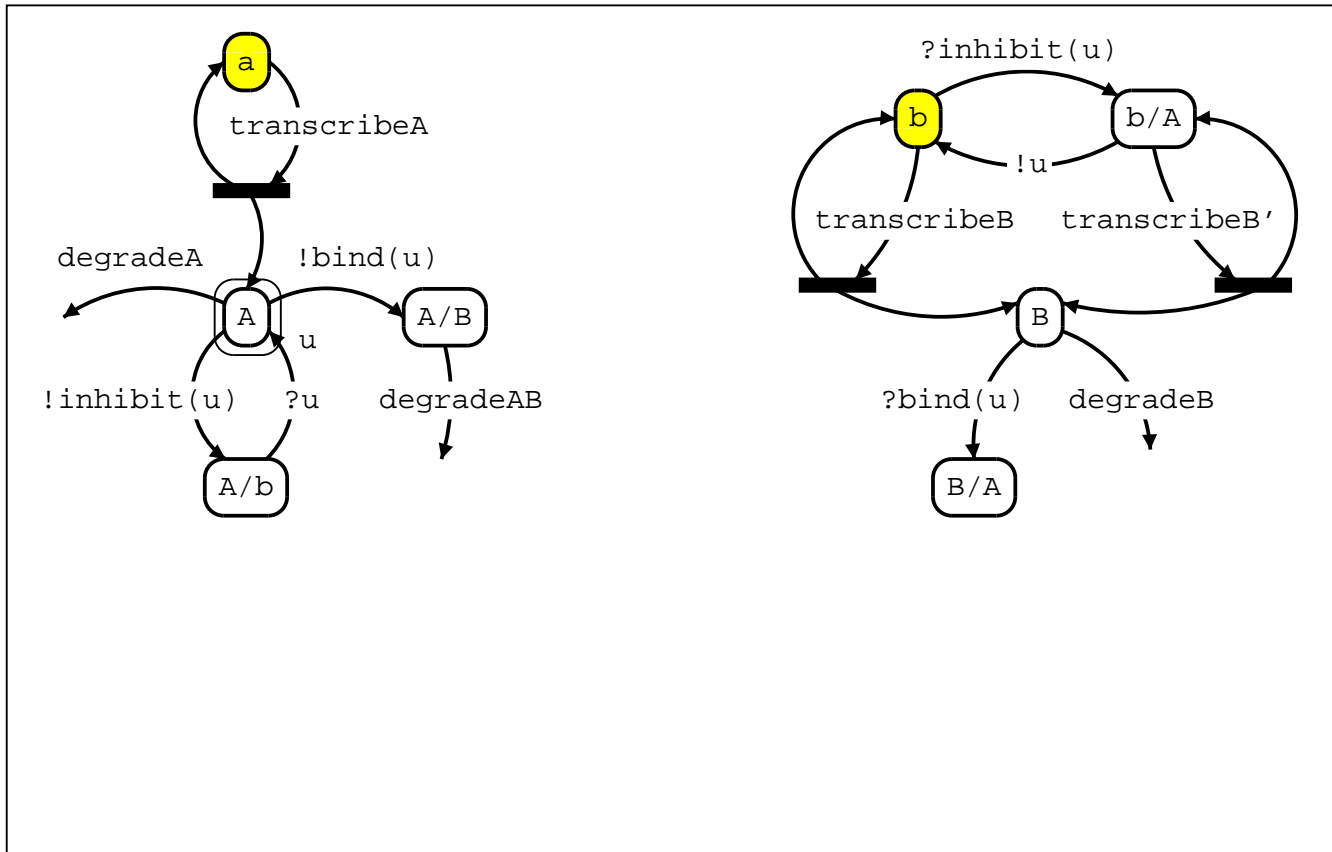
Protein *A* and *B* are irreversibly bound

# Evolved Gene Network



Complex  $AB$  can be degraded

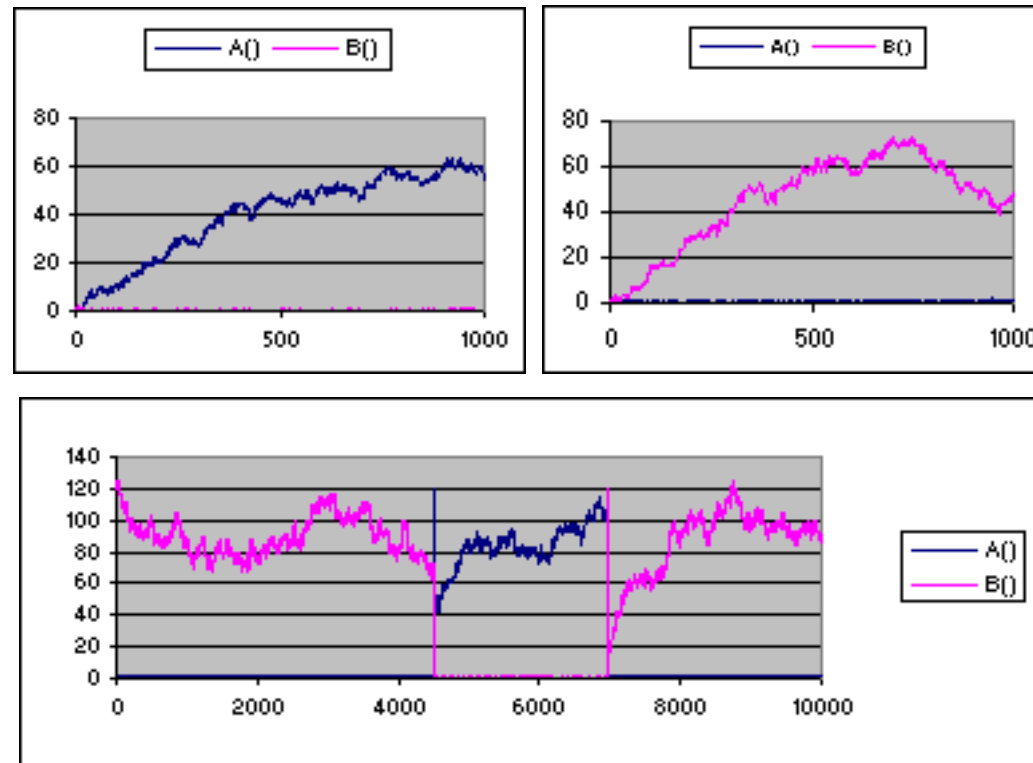
# Evolved Gene Network



Complex  $AB$  has been degraded

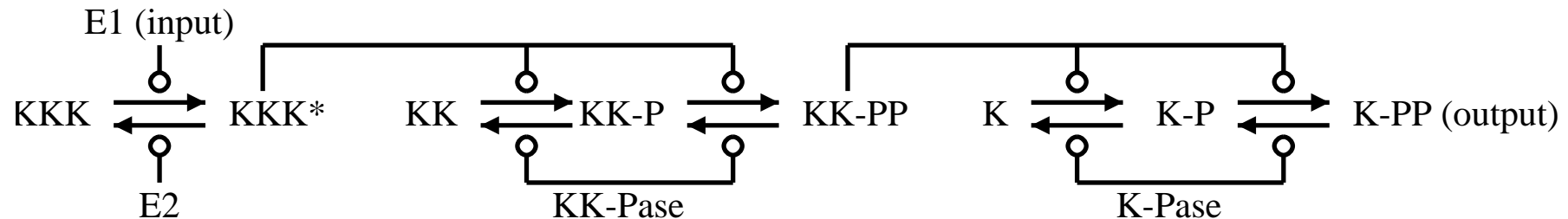
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# Evolved Gene Network: Simulation Results



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## Mapk Cascade [Huang and Ferrel, 1996]



- System originally described using a set of reaction equations
  - ❑ Converted to ordinary differential equations, which were solved numerically
  - ❑ Response curves shown to be steeply sigmoidal ( $\simeq$ Hill 5).
- System functions as follows:
  - ❑ The enzyme E1 drives the transformation from KKK to KKK\*
  - ❑ KKK\* drives the transformation from KK to KK-P to KK-PP
  - ❑ KK-PP drives the transformation from K to K-P to K-PP

## Mapk Cascade: Equations

Reaction Equation:

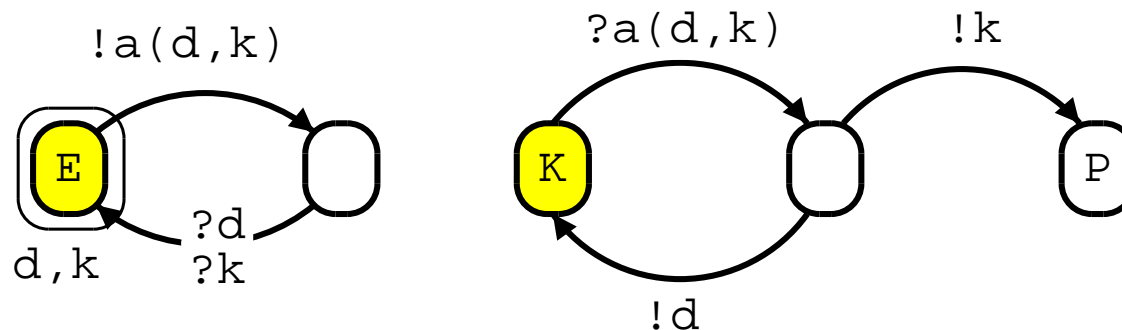


Pi-calculus Processes:

$$E(a) \triangleq \nu d \nu k !a(d, k). (?d.E(a) + ?k.E(a))$$

$$K(a) \triangleq ?a(d, k). (!d.K(a) + !k.P())$$

Graphical Representation:





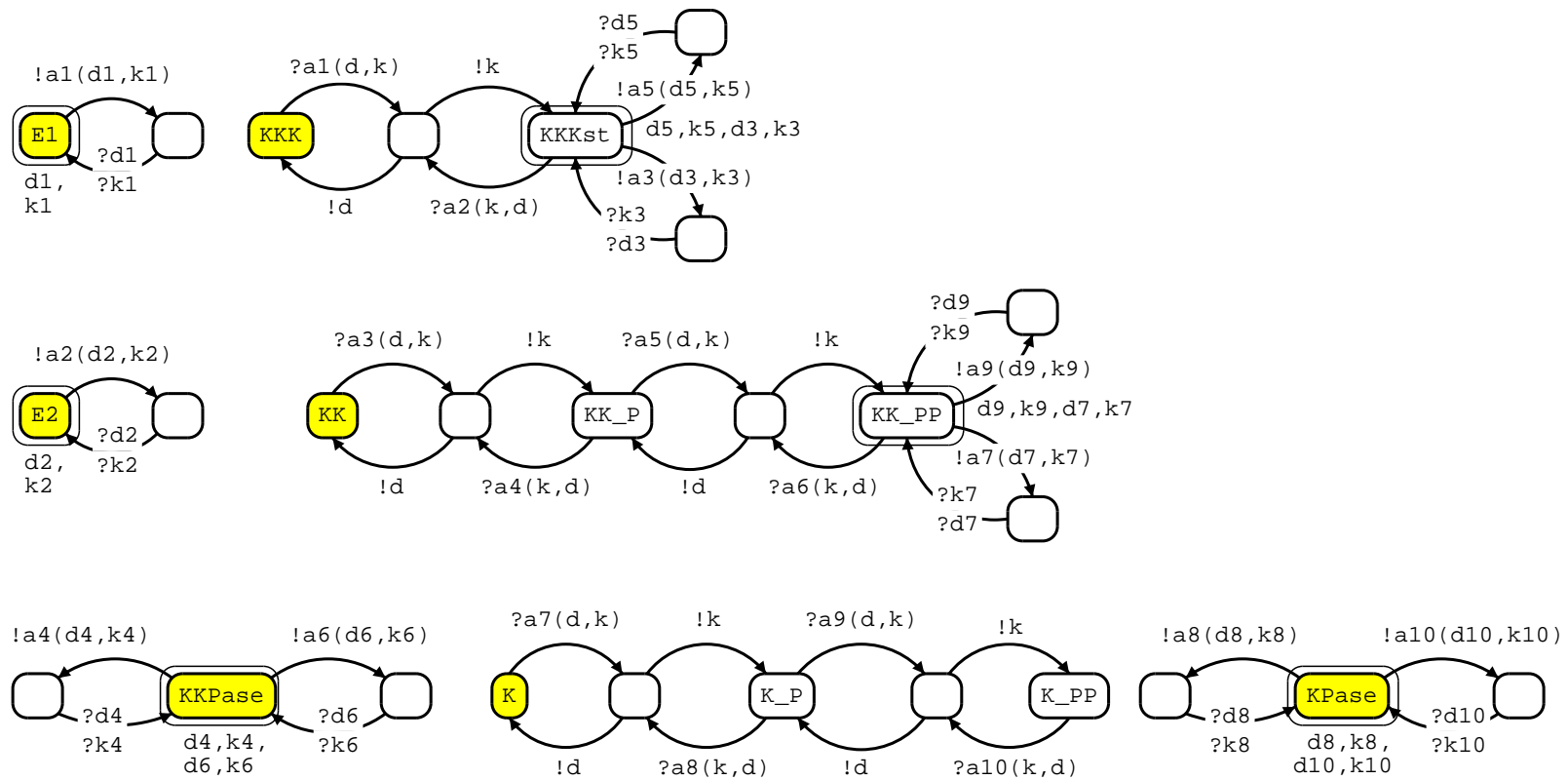
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# Mapk Cascade: SPiM Code

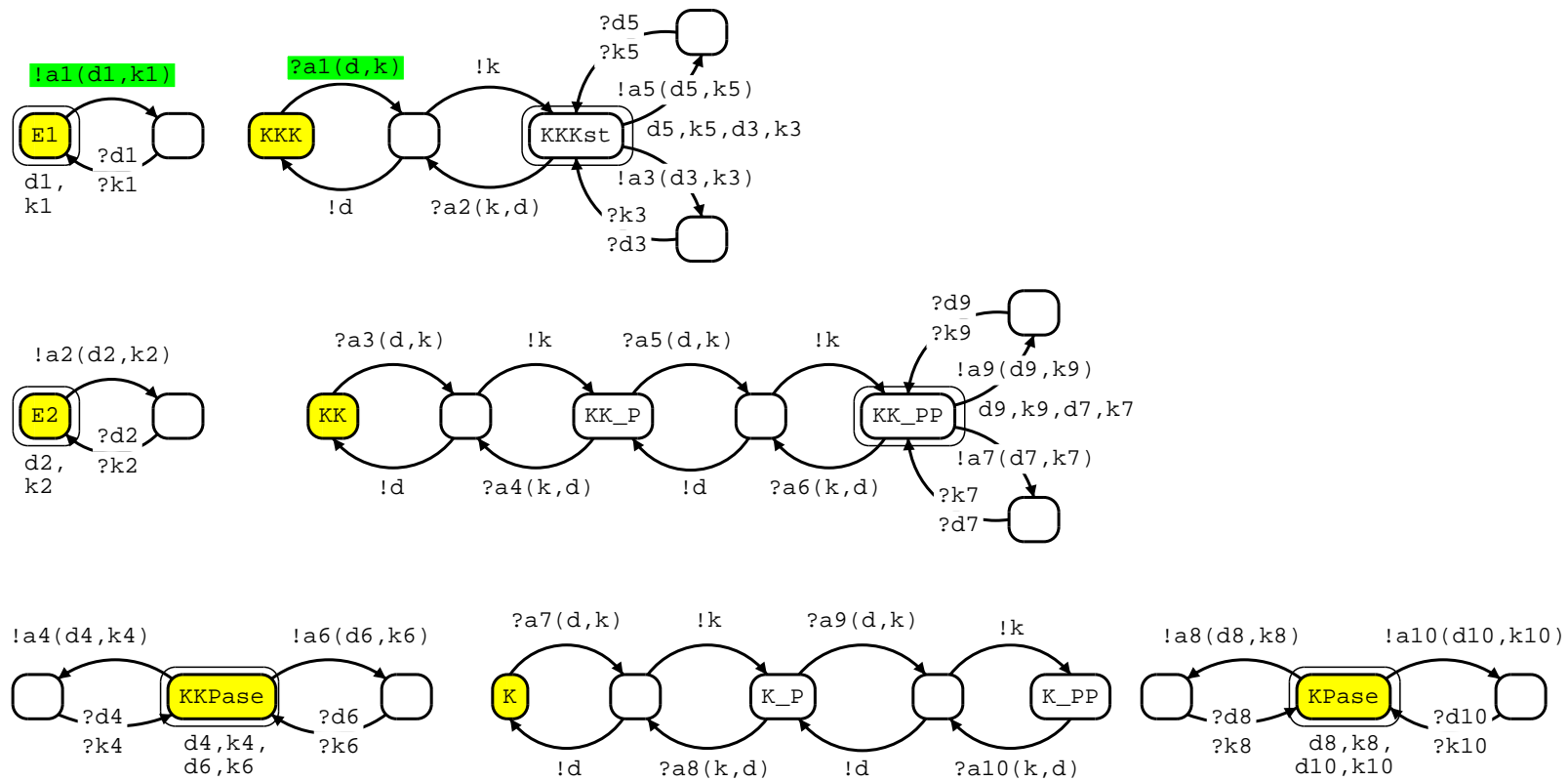
```
let E1() = (
  new k1@rk1:chan new d1@rd1:chan
  !a1(d1,k1); do ?d1;E1() or ?k1;E1()
)
let E2() = (
  new k2@rk2:chan new d2@rd2:chan
  !a2(d2,k2); do ?d2;E2() or ?k2;E2()
)
let KKK() = ?a1(d,k); KKK_E(d,k)
and KKK_E(d:chan,k:chan) =
  do !d;KKK() or !k;KKKst()
and KKKst() = (
  new d3@rd3:chan new k3@rk3:chan
  new d5@rd5:chan new k5@rk5:chan
  do ?a2(k,d); KKK_E(d,k)
  or !a3(d3,k3); (do ?d3;KKKst() or ?k3;KKKst())
  or !a5(d5,k5); (do ?d5;KKKst() or ?k5;KKKst())
)
let KK() = ?a3(d,k); KK_E(d,k)
and KK_E(d:chan,k:chan) = do !d;KK() or !k;KK_P()
and KK_P() =
  do ?a4(k,d);KK_E(d,k) or ?a5(d,k);KK_P_E(d,k)
and KK_P_E(d:chan,k:chan) =
  do !d;KK_P() or !k;KK_PP()
and KK_PP() = (
  new d7@rd7:chan new k7@rk7:chan
  new d9@rd9:chan new k9@rk9:chan
  do ?a6(k,d); KK_P_E(d,k)
  or !a7(d7,k7); (do ?d7;KK_PP() or ?k7;KK_PP())
  or !a9(d9,k9); (do ?d9;KK_PP() or ?k9;KK_PP())
)
let K() = ?a7(d,k); K_E(d,k)
and K_E(d:chan,k:chan) = do !d; K() or !k; K_P()
and K_P() = do ?a8(k,d);K_E(d,k) or ?a9(d,k);K_P_E(d,k)
and K_P_E(d:chan,k:chan) = do !d;K_P() or !k;K_PP()
and K_PP() = ?a10(k,d); K_P_E(d,k)

let KKPase() = (
  new d4@rd4:chan new k4@rk4:chan
  new d6@rd6:chan new k6@rk6:chan
  do !a4(d4,k4); (do ?d4;KKPase() or ?k4;KKPase())
  or !a6(d6,k6); (do ?d6;KKPase() or ?k6;KKPase())
)
let KPase() = (
  new d8@rd8:chan new k8@rk8:chan
  new d10@rd10:chan new k10@rk10:chan
  do !a8(d8,k8); (do ?d8;KPase() or ?k8;KPase())
  or !a10(d10,k10); (do ?d10;KPase() or ?k10;KPase())
)
run 100 of (KKK() | KK() | K())
run ( E2() | KKPase() | KPase() | E1())
```

# Mapk Cascade



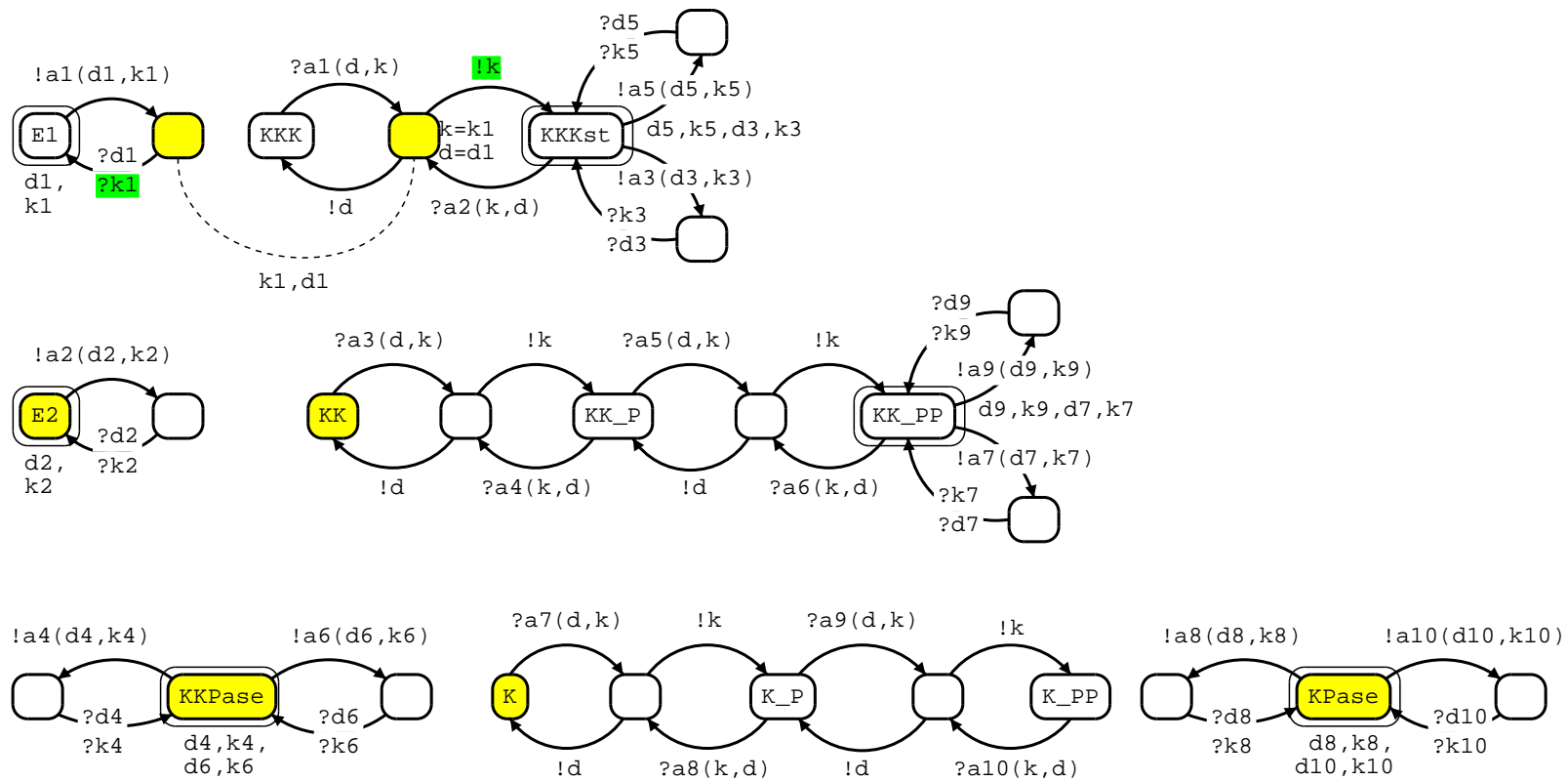
# Mapk Cascade



Enzyme  $E_1$  can bind to substrate KKK using channel  $a_1$

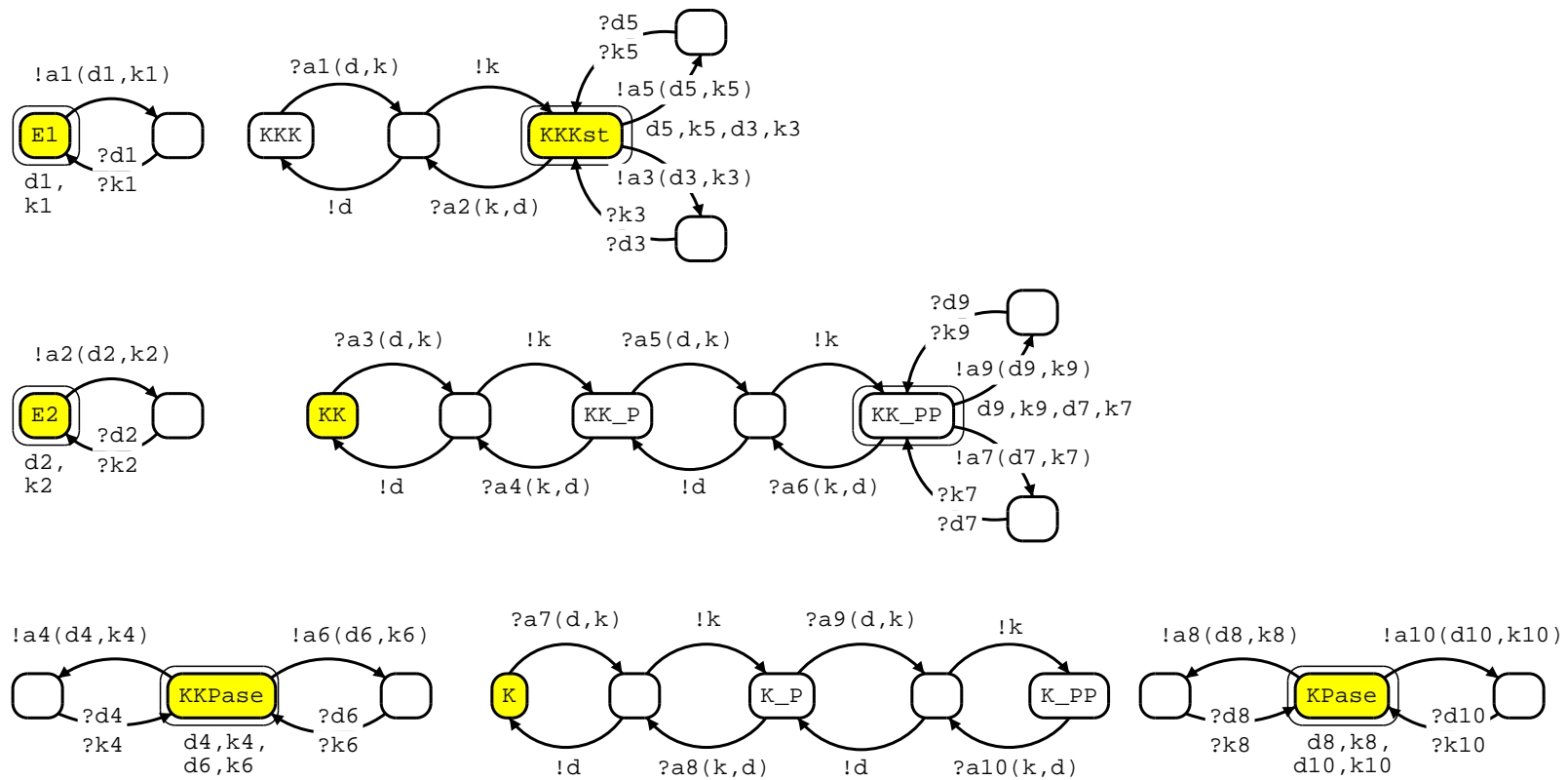


# Mapk Cascade



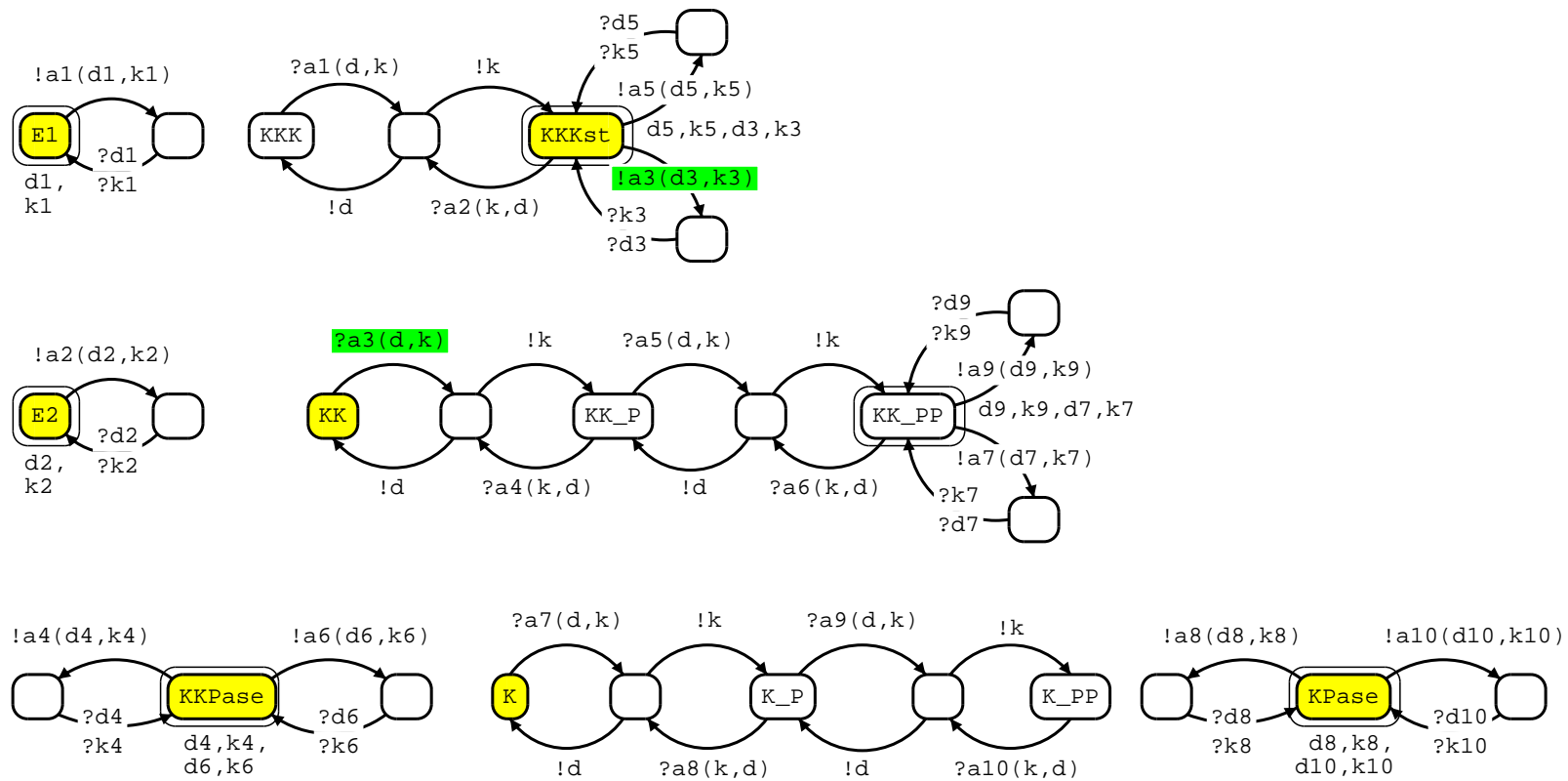
$E_1$  can react with KKK using channel  $k_1$

# Mapk Cascade



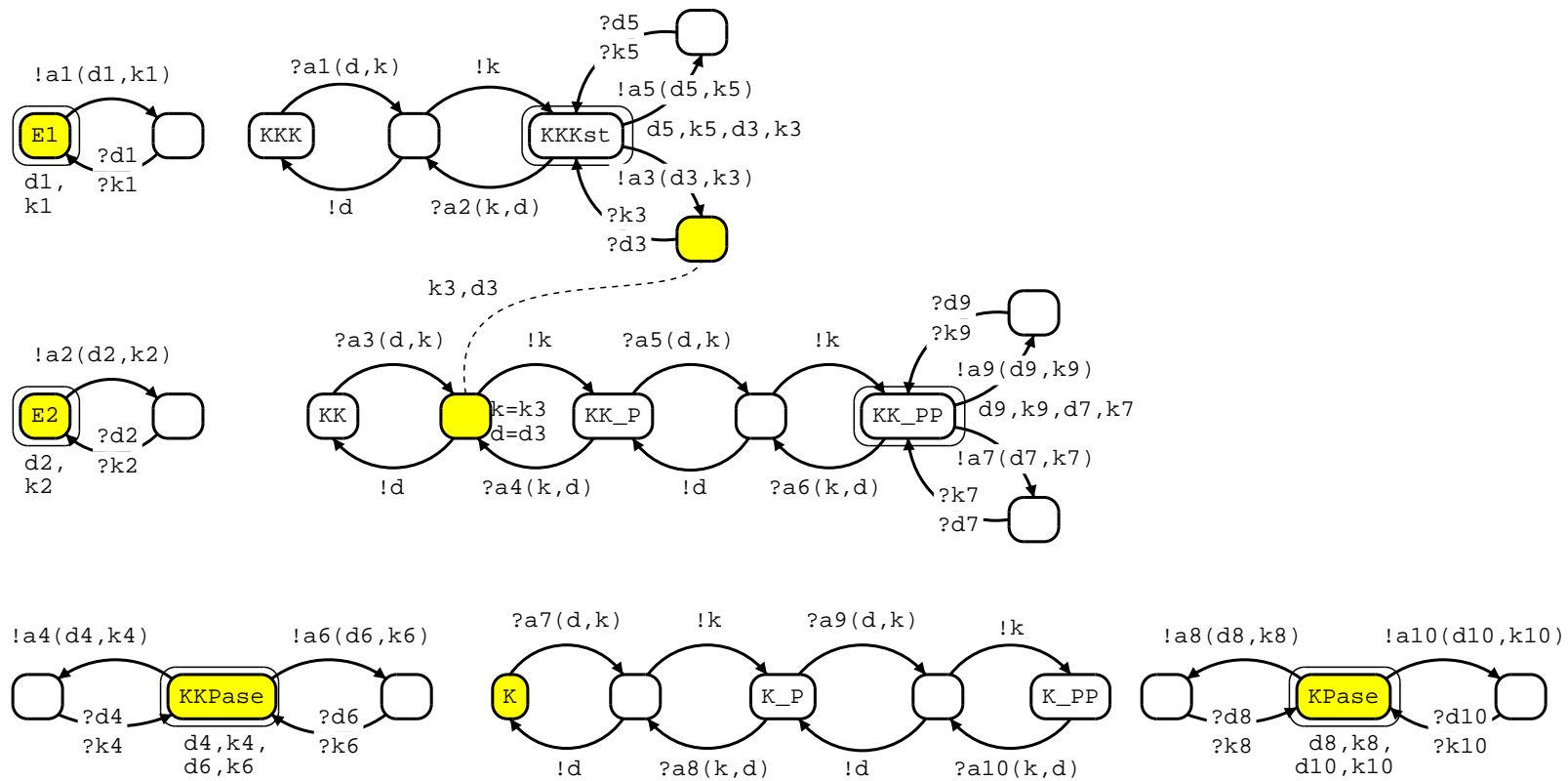
KKK is transformed to KKK\*

# Mapk Cascade



KKK\* can bind with KK using channel  $a_3$

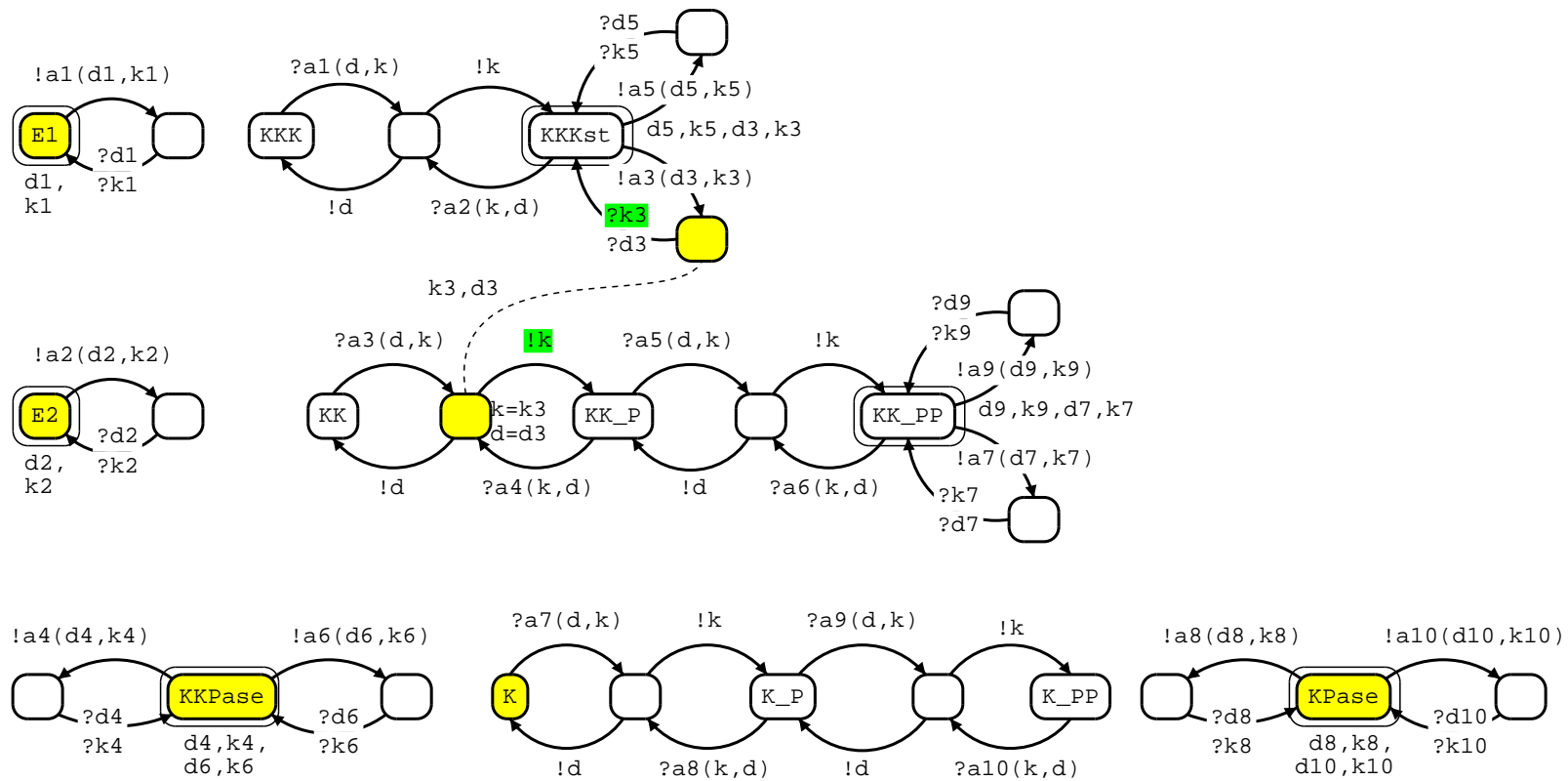
# Mapk Cascade



$KKK^*$  is bound to  $KK$  by private channels  $d_3$  and  $k_3$

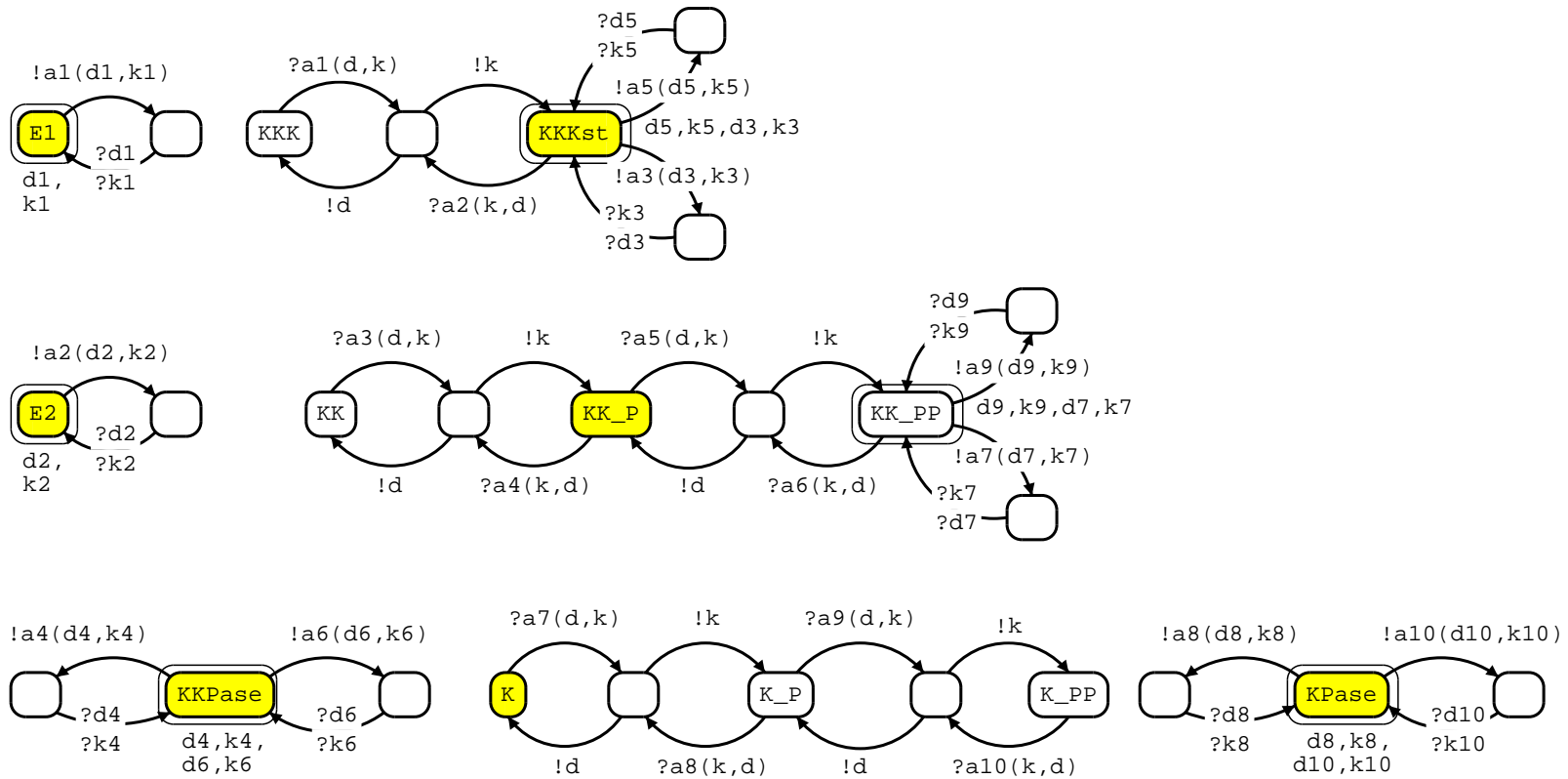


# Mapk Cascade



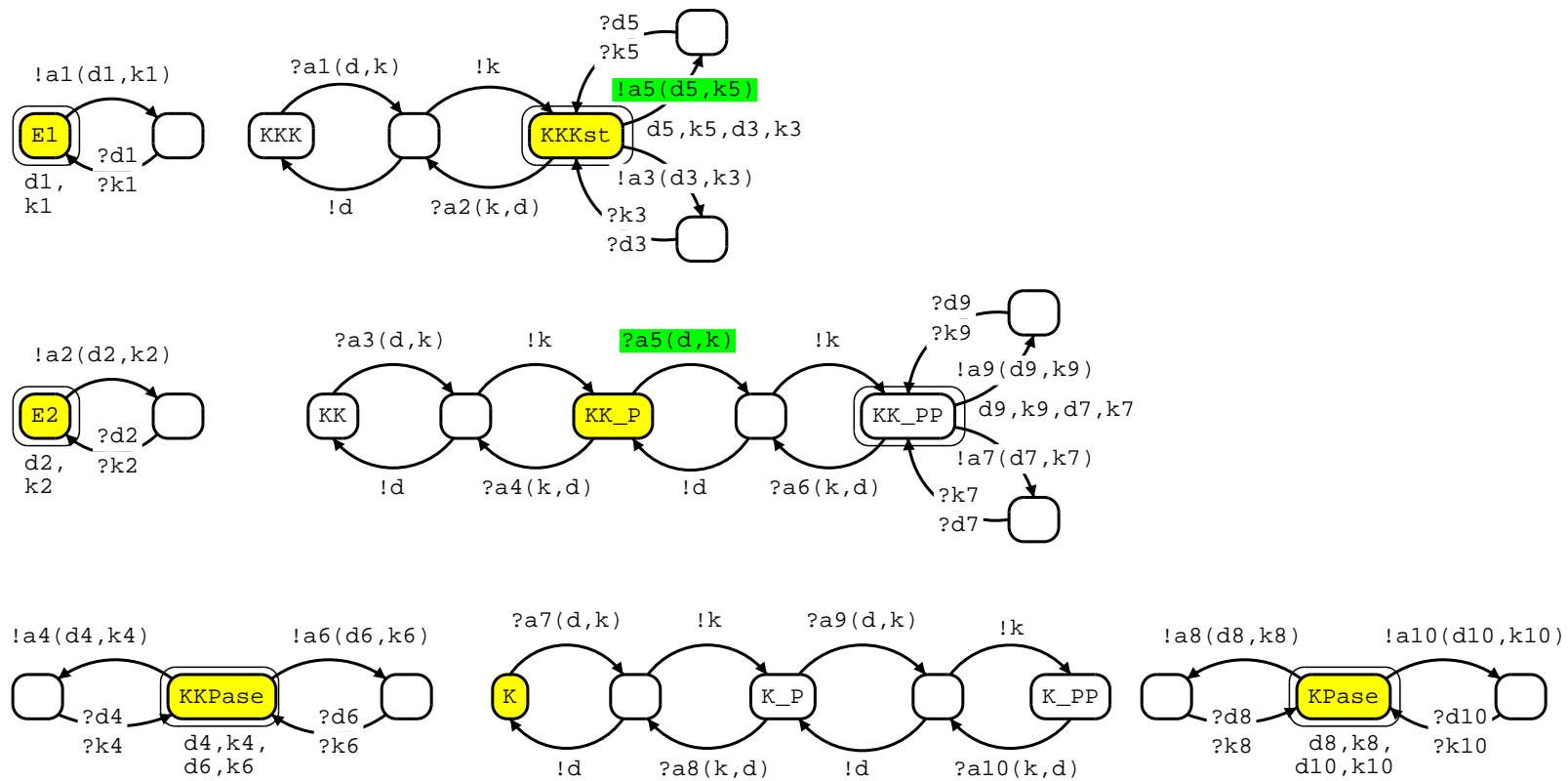
KKK\* can react with KK using channel  $k_3$

# Mapk Cascade



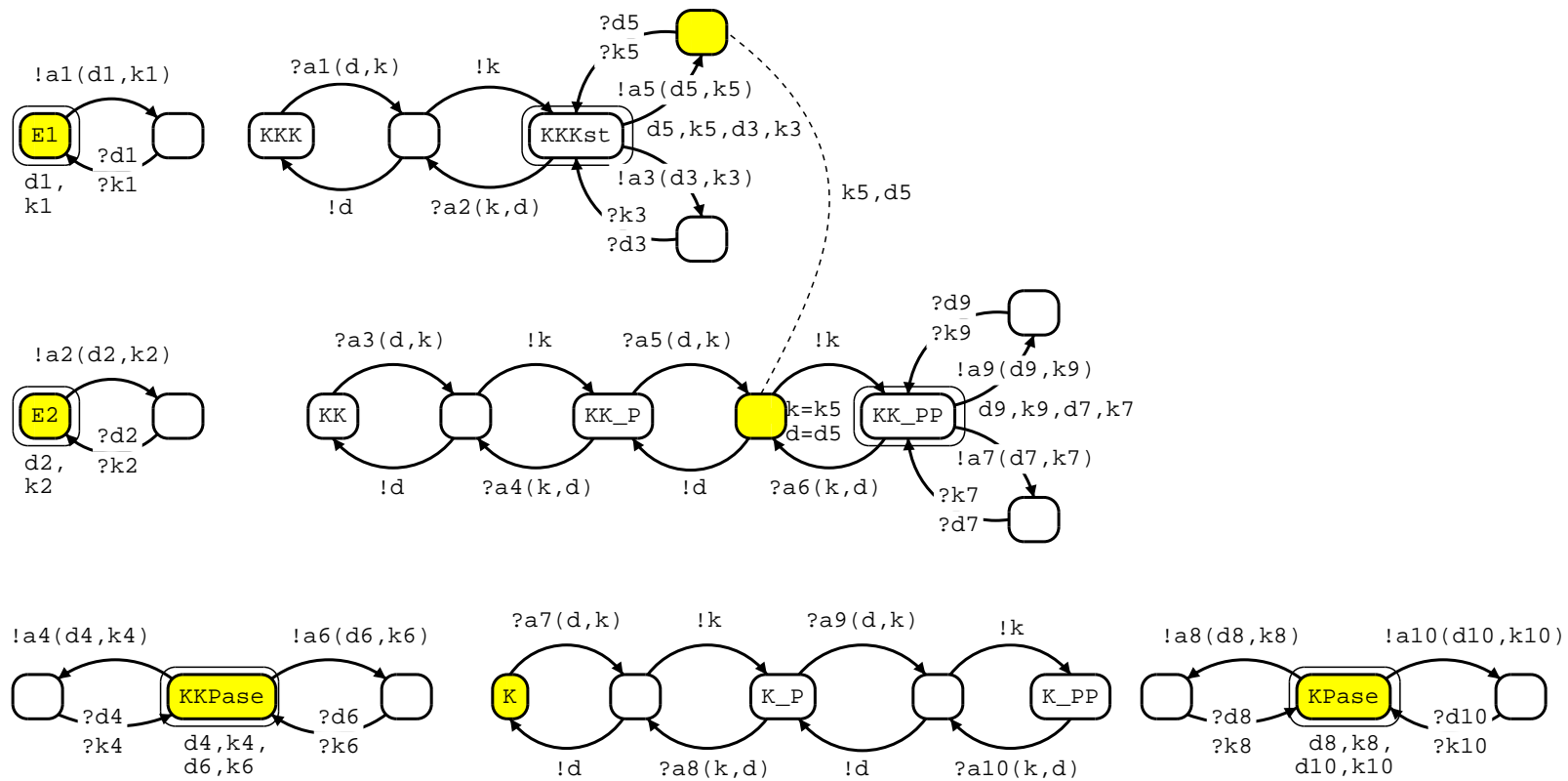
KK is transformed to KK-P

# Mapk Cascade



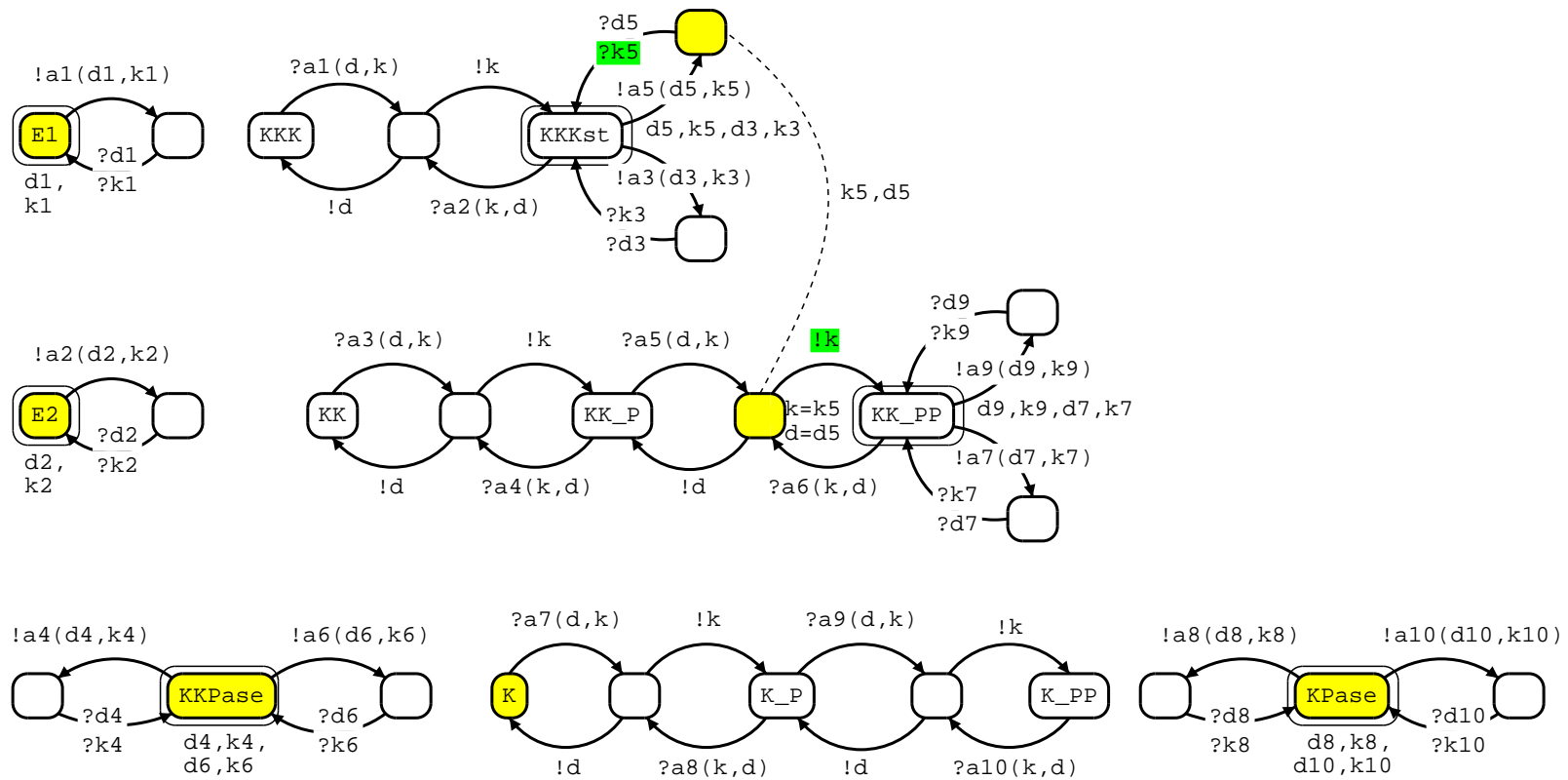
KKK\* can bind to KK-P using channel  $a_5$

# Mapk Cascade



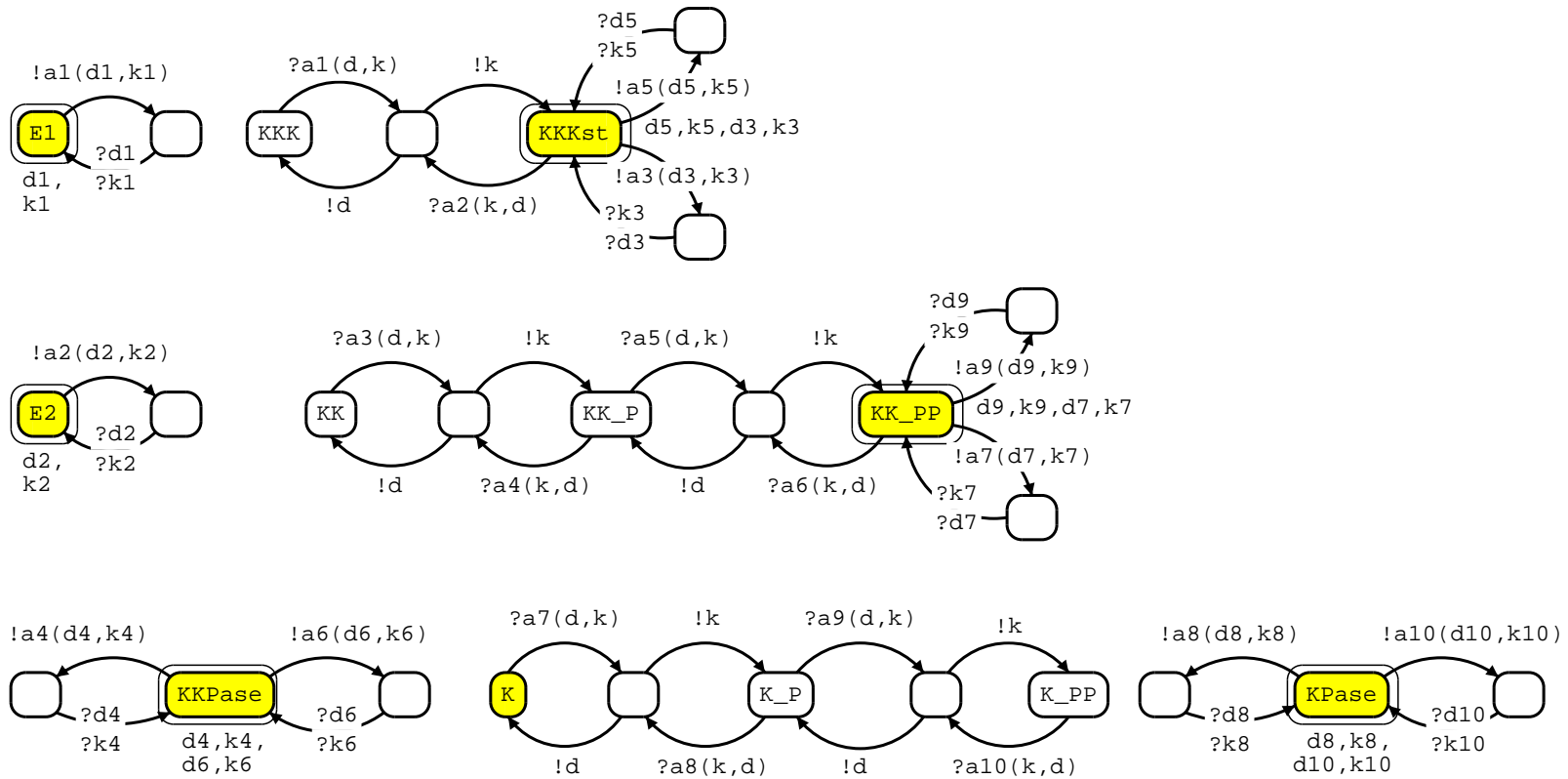
KKK\* is bound to KK-P by channels  $d_5$  and  $k_5$

# Mapk Cascade



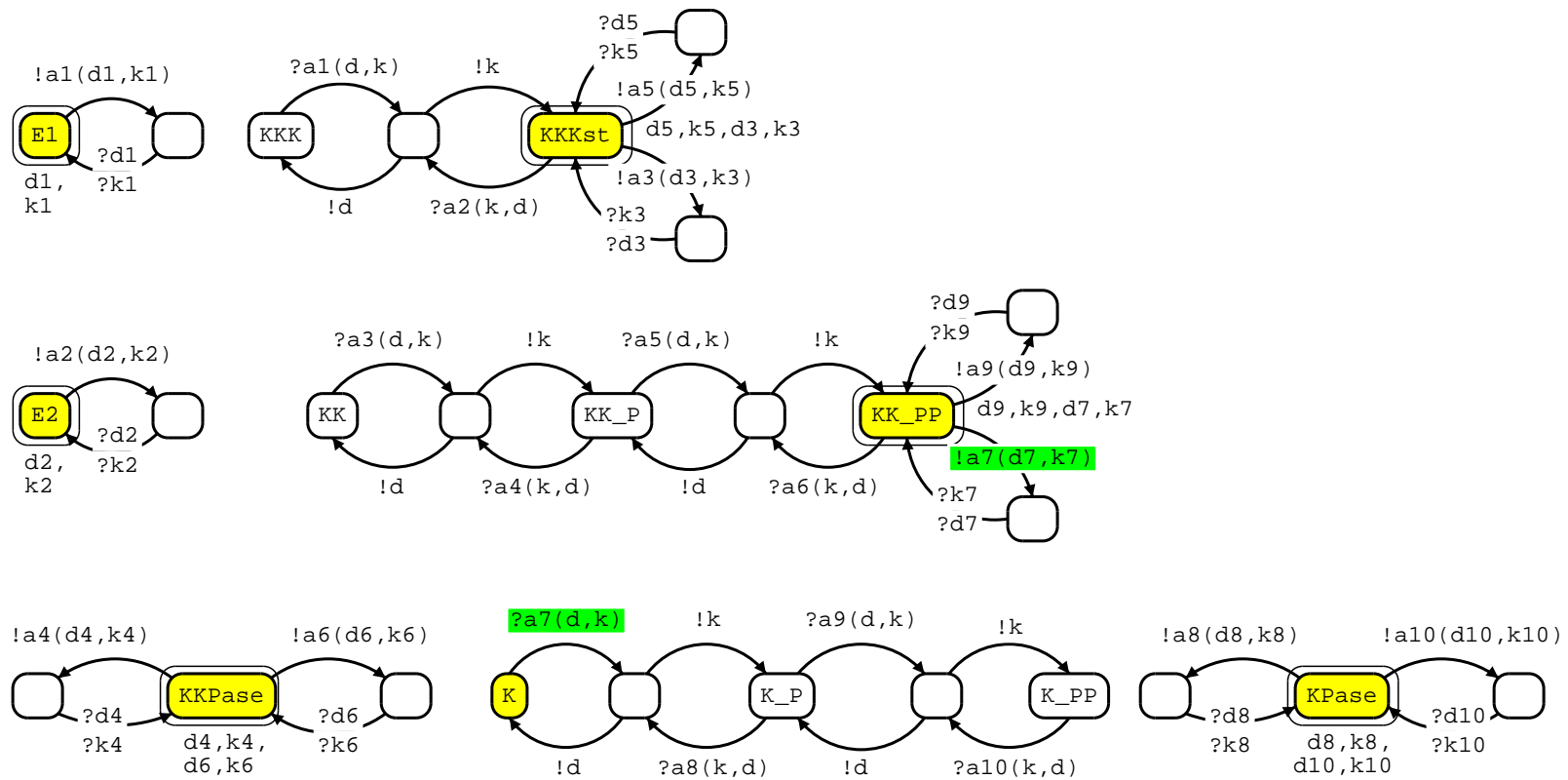
KKK\* can react with KK-P using channel  $k_5$

# Mapk Cascade



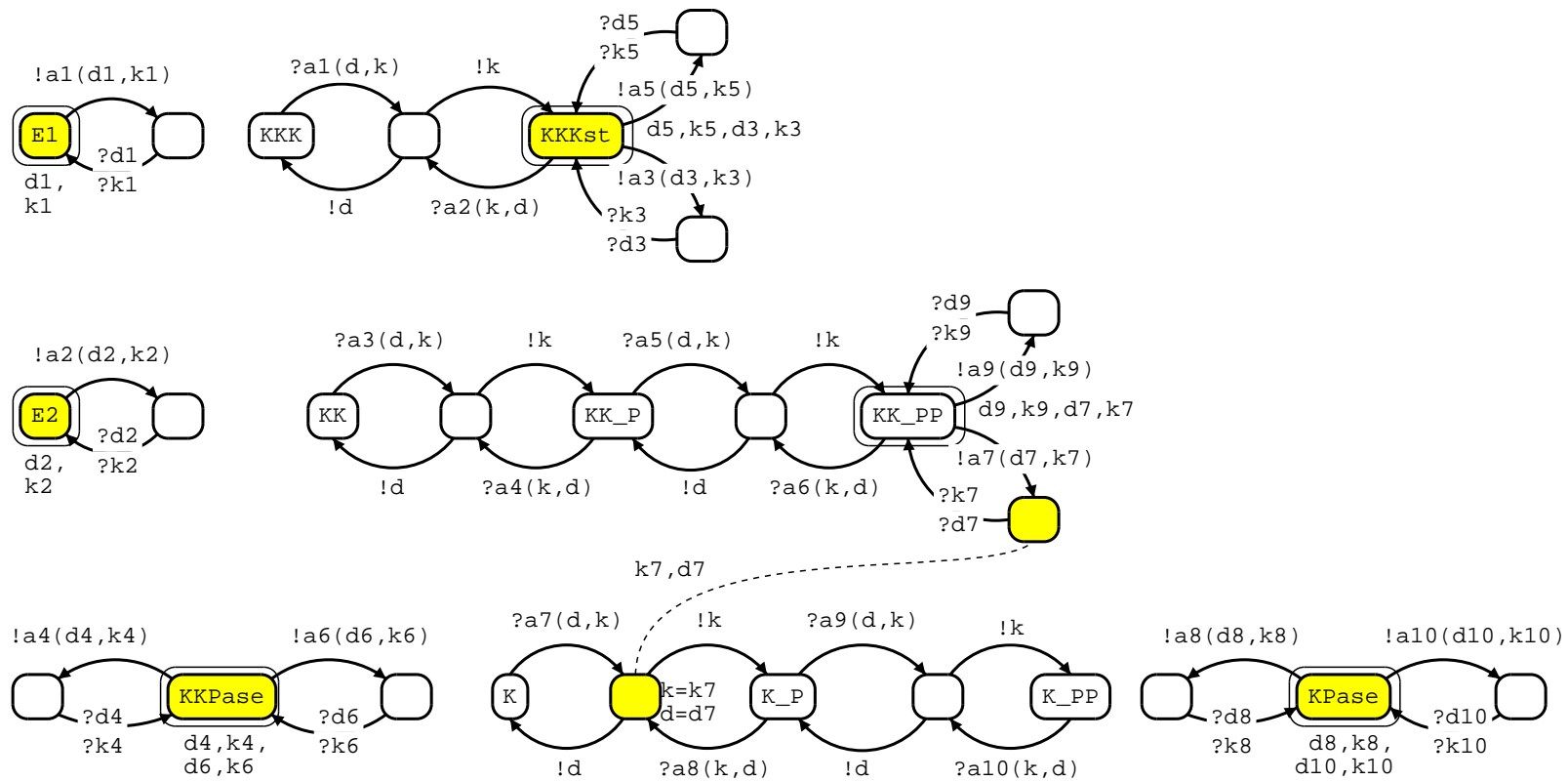
KK-P is transformed to KK-PP

# Mapk Cascade



KK-PP can bind to K using channel  $a_7$

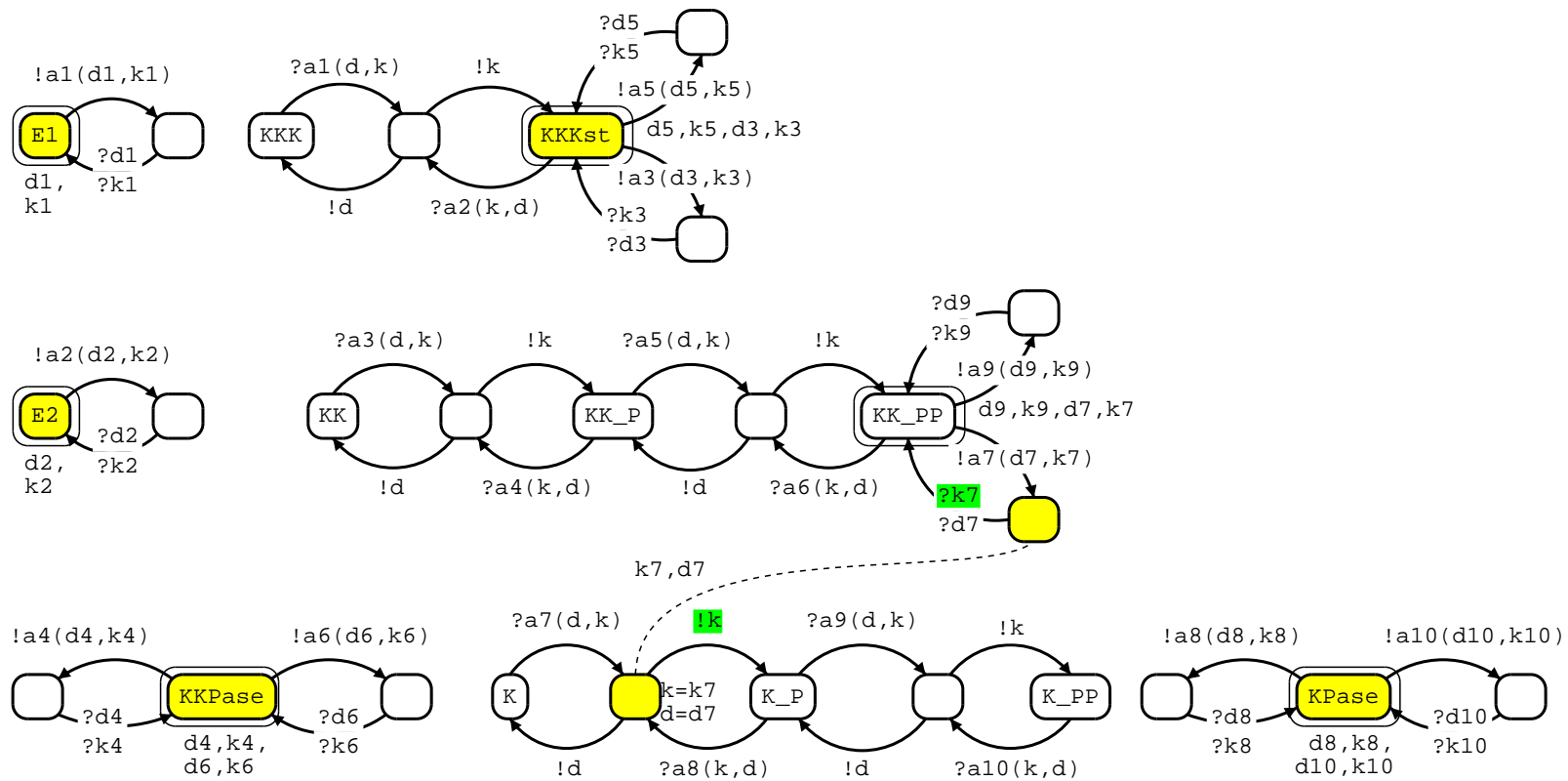
# Mapk Cascade



KK-PP is bound to K by channels  $d_7$  and  $k_7$

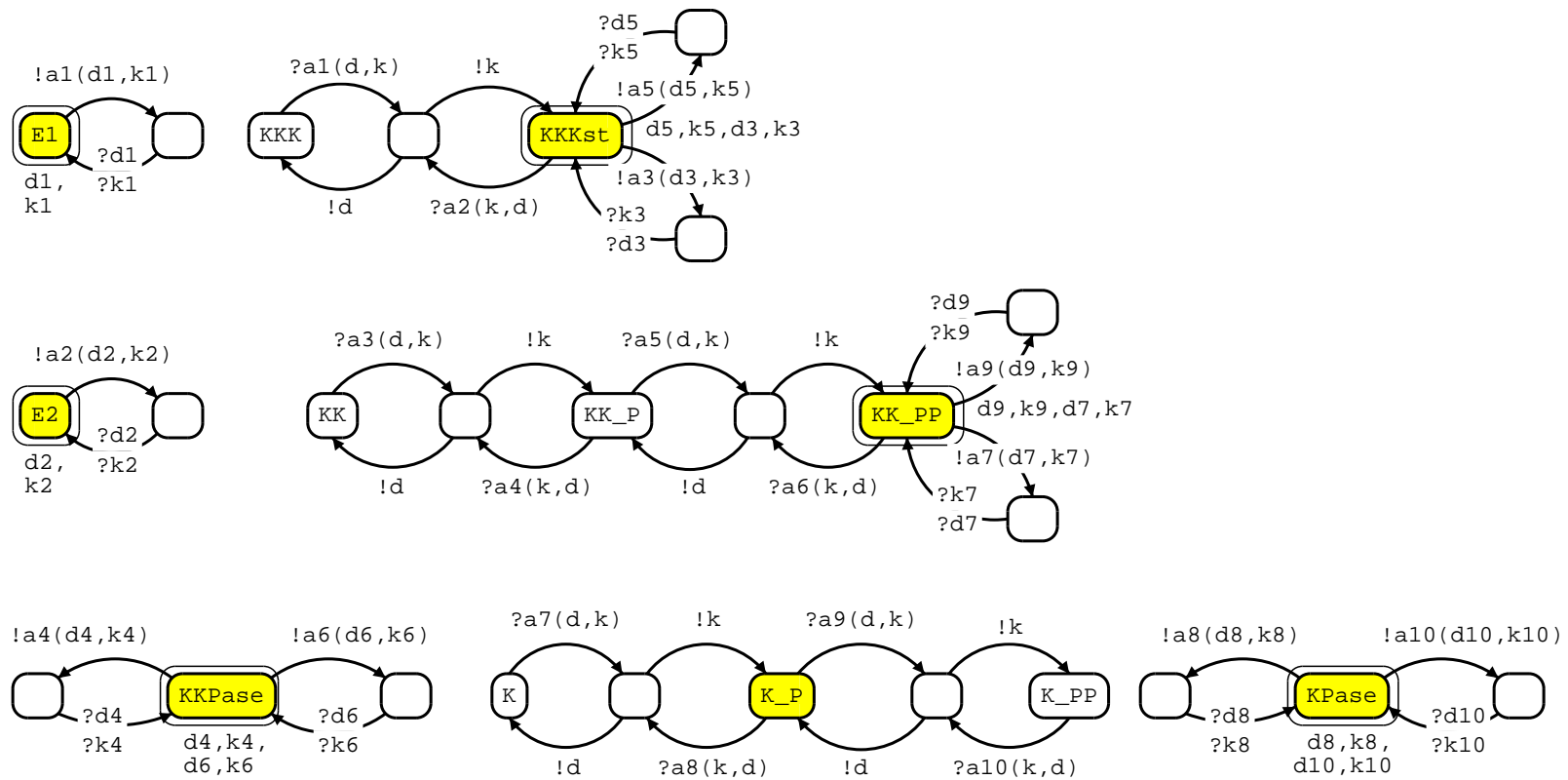


# Mapk Cascade



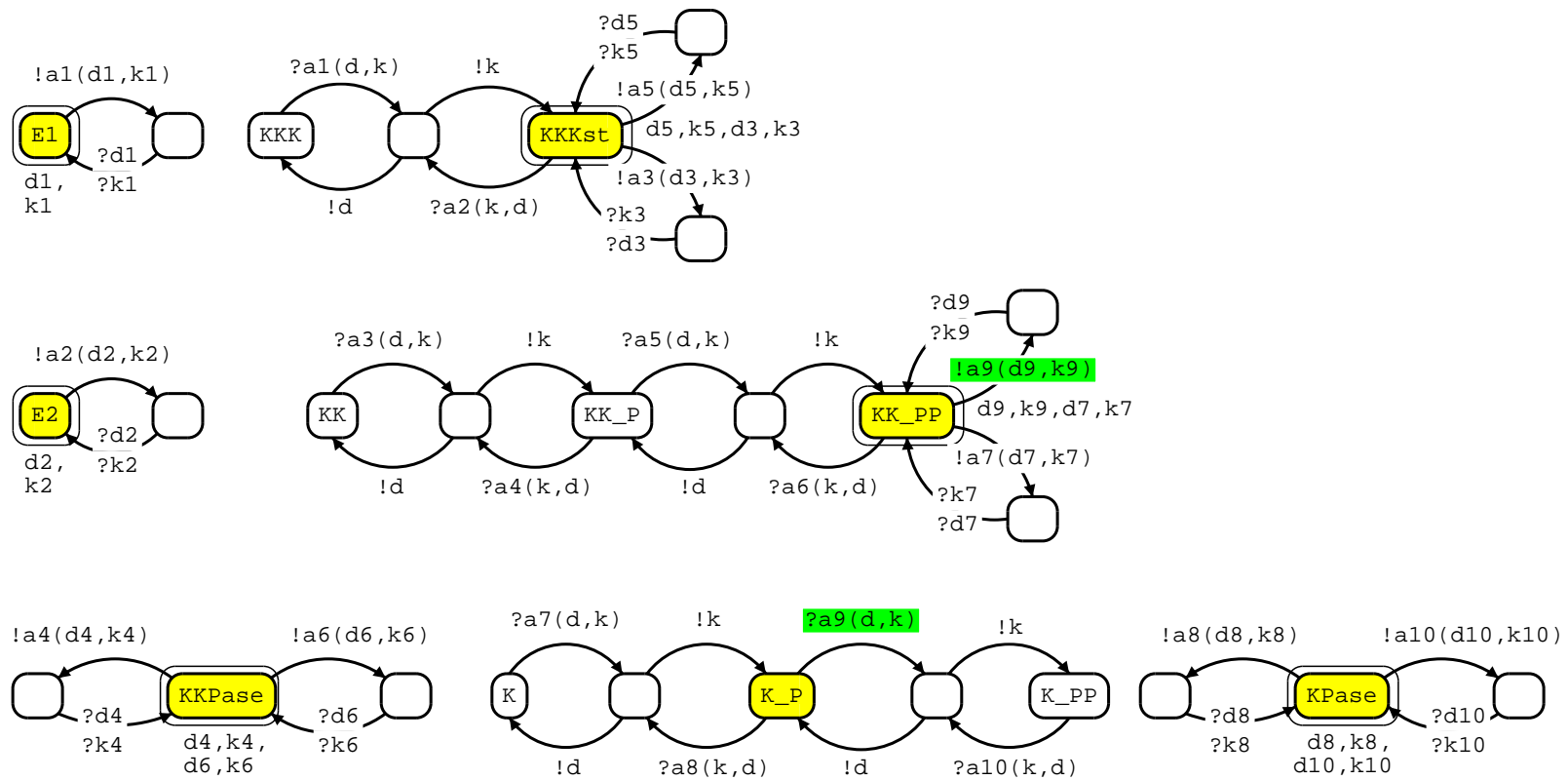
KK-PP can react with K using channel  $k_7$

# Mapk Cascade



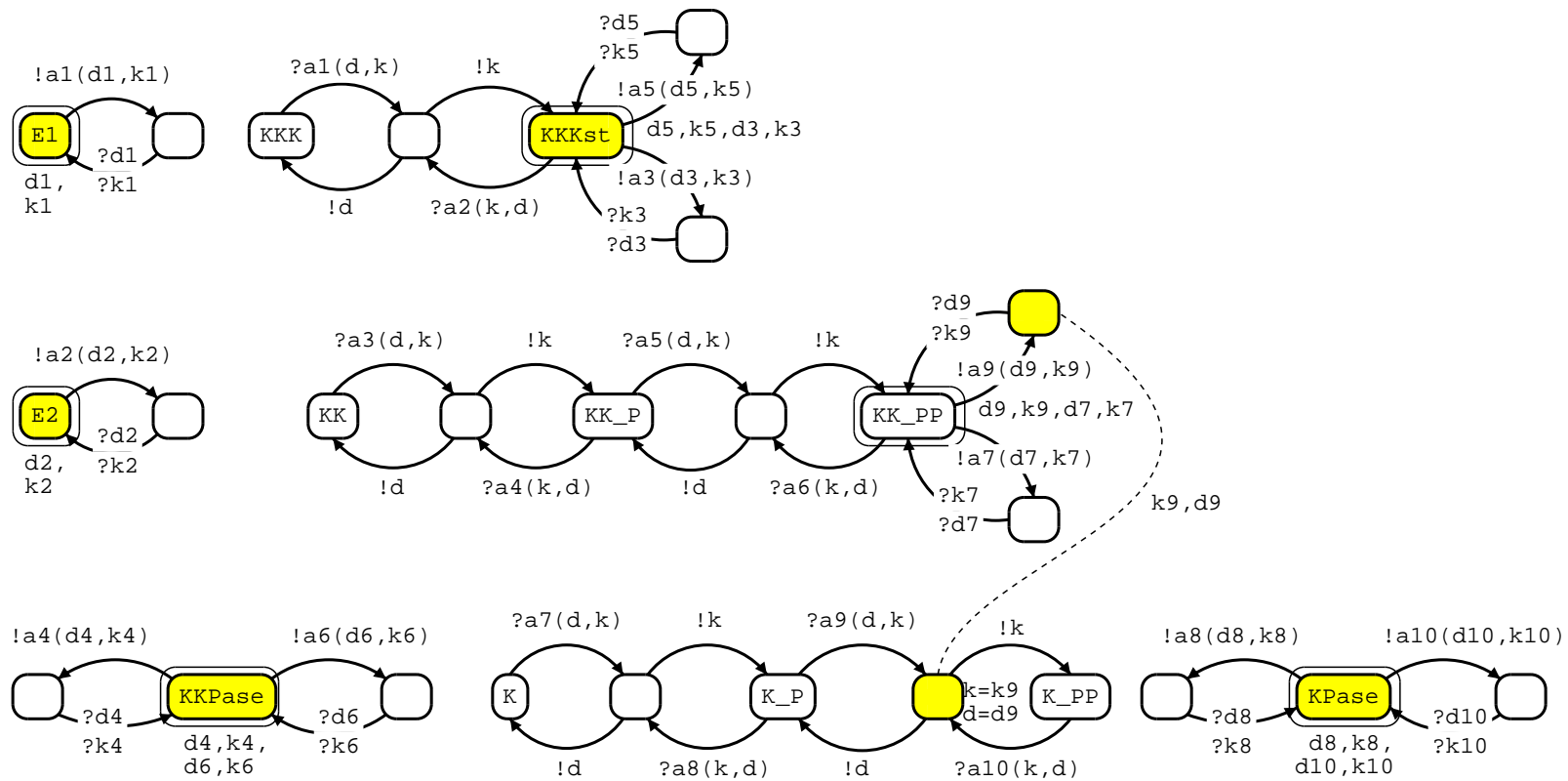
K is transformed to KK-P

# Mapk Cascade



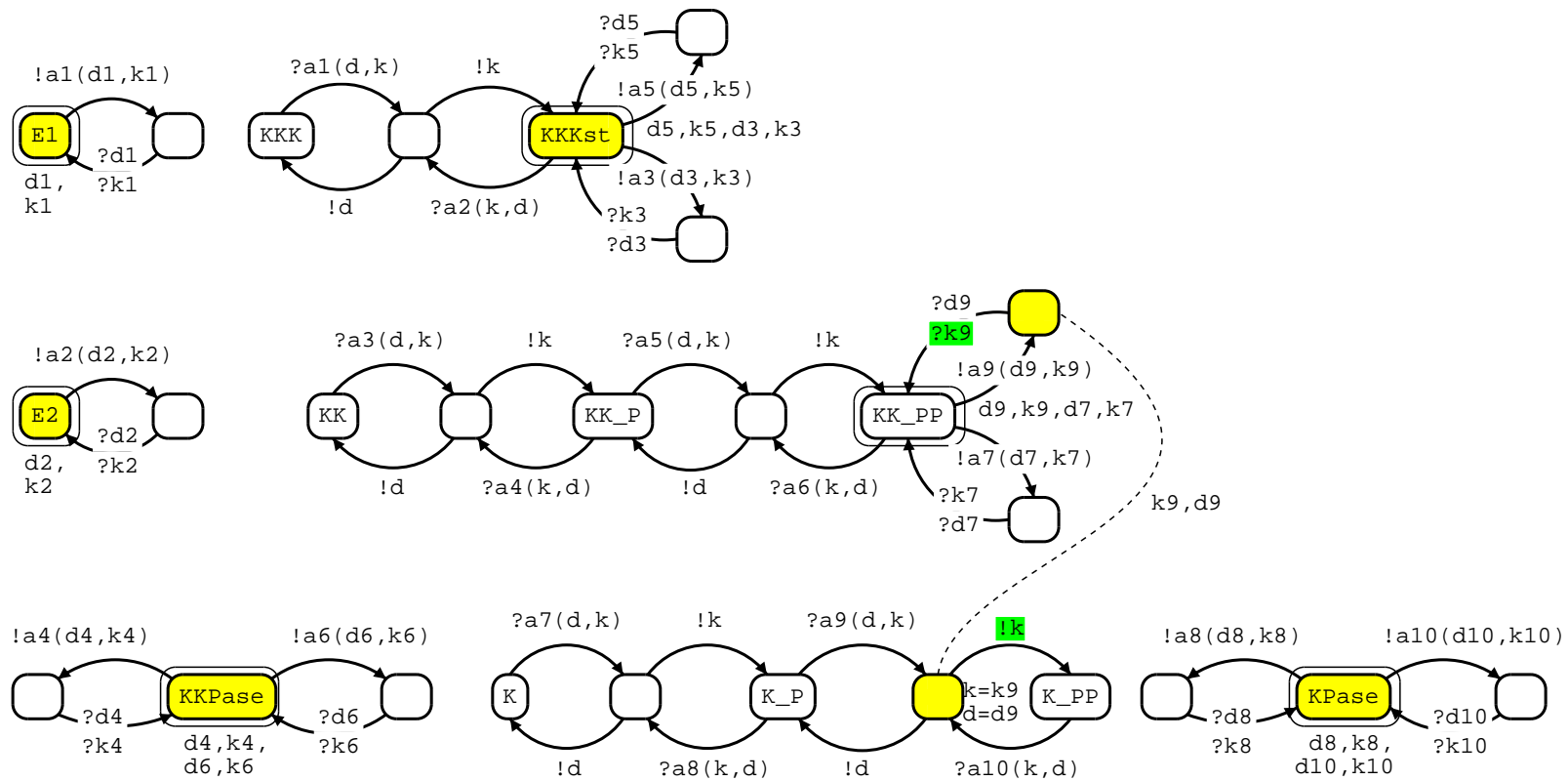
KK-PP can bind to KK-P using channel  $a_9$

# Mapk Cascade



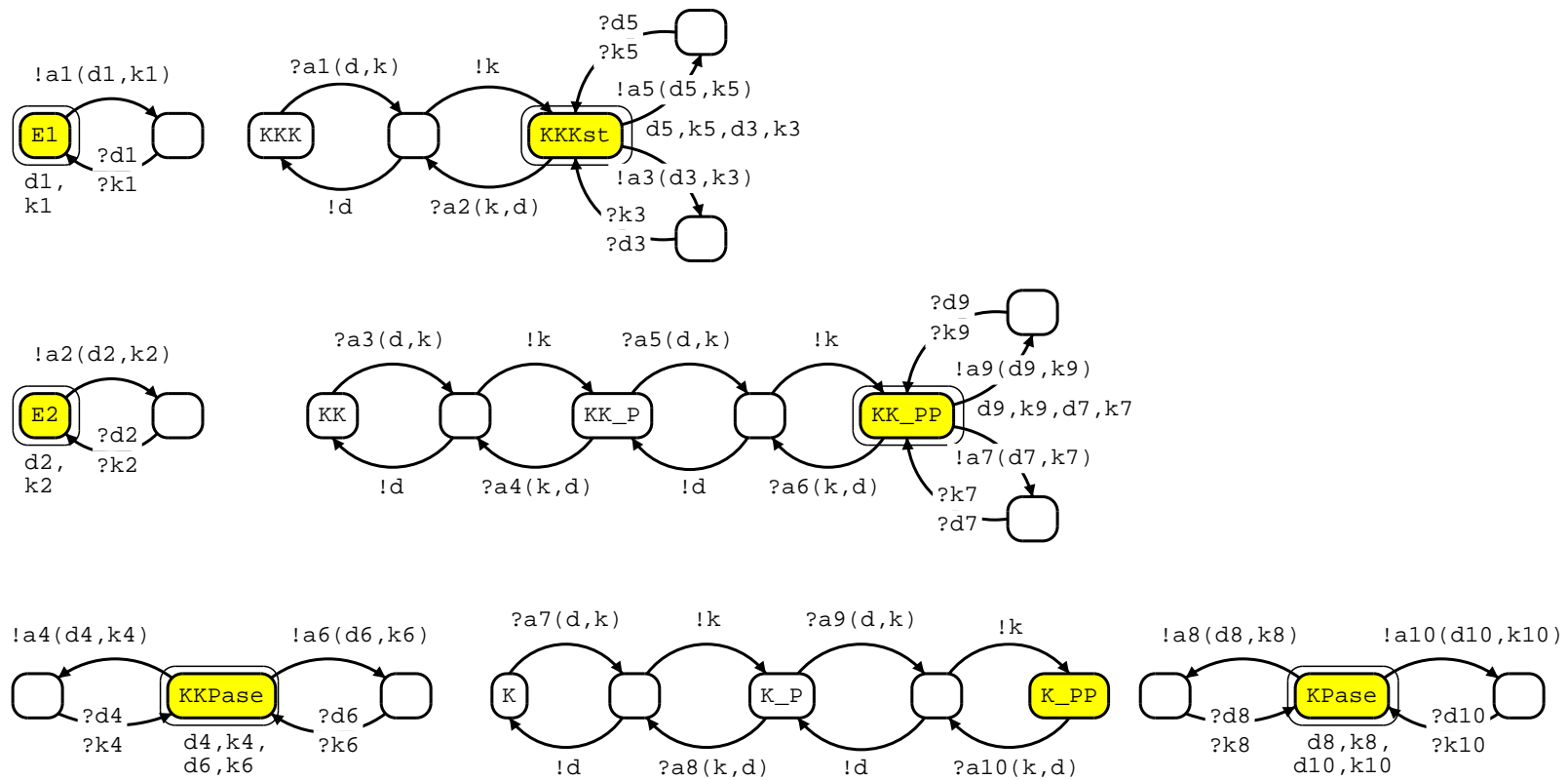
KK-PP is bound to KK-P by channels  $d_9$  and  $k_9$

# Mapk Cascade



KK-PP can react with K-P using channel  $k_9$

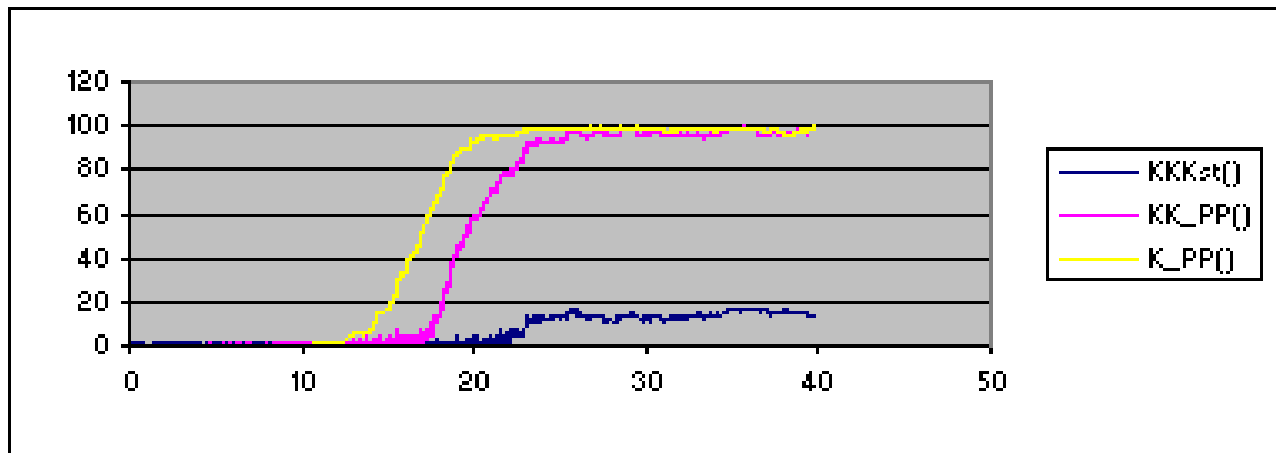
# Mapk Cascade



K-P is transformed to K-PP, completing the cascade

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## Mapk Cascade: Results



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## Open Graph Syntax (DOT)

$G ::=$

- $I[label]$       Labelled node with id  $I$
- |  $I \xrightarrow{label}_{label} J$       Labelled edge from node  $I$  to node  $J$
- |  $G; G'$       Sequence of graph declarations



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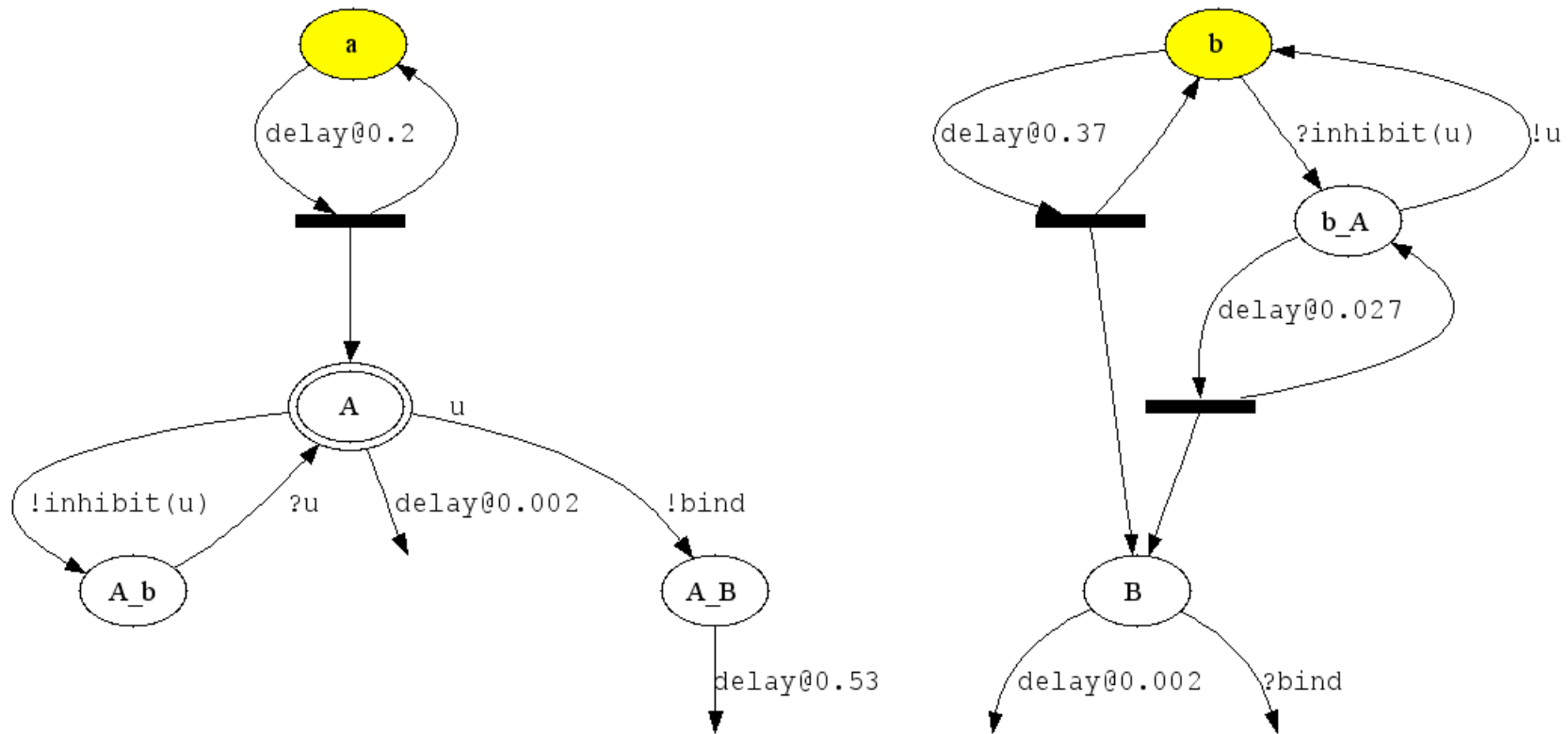
## Graph Generation

**Proposition 2.**  $\forall \Gamma. \Gamma \in \text{GSPi}_\Gamma \Rightarrow (|\Gamma|)_I \in \text{DOT}$

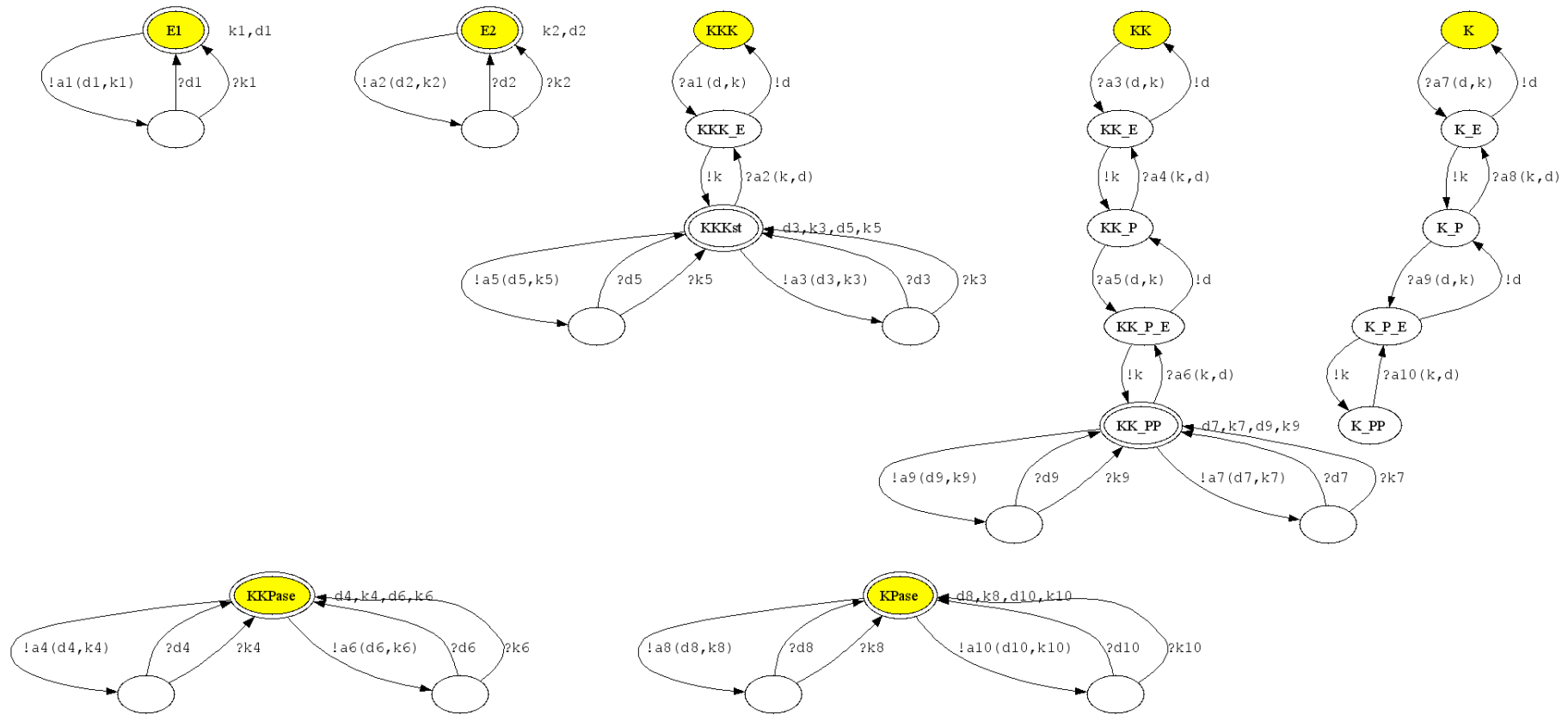
$$\begin{aligned}
 (|\emptyset|)_I &\triangleq \emptyset \\
 (|X(\vec{m}) \triangleq C, \Gamma|)_I &\triangleq (|C|)_X; (|\Gamma|)_I \\
 (|\nu x_1 \dots \nu x_N C|)_I &\triangleq I[x_1, \dots, x_N]; (|C|)_I \\
 X(\vec{m}) \triangleq C \quad \Rightarrow \quad (|X(\vec{n})|)_I &\triangleq I \longrightarrow_{\{\vec{n}/\vec{m}\}} X \\
 X(\vec{m}) \triangleq C \quad \Rightarrow \quad (|X(\vec{n}) \mid \Pi|)_I &\triangleq I \longrightarrow_{\{\vec{n}/\vec{m}\}} X; (|\Pi|)_I \\
 (|\mathbf{0}|)_I &\triangleq \emptyset \\
 X(\vec{m}) \triangleq C \quad \Rightarrow \quad (|\pi.X(\vec{n}) + \Sigma|)_I &\triangleq I \xrightarrow{\pi}_{\{\vec{n}/\vec{m}\}} X; (|\Sigma|)_I
 \end{aligned}$$

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## Generated Graphs: Evolved Network



# Generated Graph: Mapk Cascade



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## Related Work

- Statecharts [Harel, 1987] highlighted the need for a scalable, self-contained graphical representation of concurrent systems.
- Synchronous variant to Statecharts allows concurrent processes to synchronise on shared labels [Andre, 1995].
- Foundational graphical representations for pi-calculus use elaborate graph rewriting rules [Milner, 1994].
- More recently, HDA [Montanari and Pistore, 2005] describes an automata-based representation for the pi-calculus.
- Preliminary informal ideas on a graphical representation for the stochastic pi-calculus in [Phillips and Cardelli, 2004].

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## Conclusion

- Presented a graphical representation for the stochastic pi-calculus.
- Used to model a Mapk signalling cascade and an evolved gene network.
- Highlights the existence of cycles, which are key to many biological systems.
- Able to animate interactions between biological system components.
- Able to clarify the overall system function and to debug changes in the system.

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## Future Work

- Observation: within a collection of mutually recursive definitions, applied arguments often the same as the formal parameters.
- Investigate additional design patterns to improve modelling and visualisation techniques.
- Use graph generation tool to implement a graphical debugger for SPiM.
- Develop a tool for drawing graphical models, which automatically generates SPiM code.
- Make modelling and simulation of biological systems more accessible to non computer scientists.

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